Value of Reverse Factoring in Multi-stage Supply Chains

Abstract: We present a mathematical model for integration, analysis, and optimization of operational and financial processes within a supply chain. Specifically, we consider commercial transactions of a large corporate customer with a small- or medium-sized supplier. We show how application of reverse factoring influences the operational and financial decisions of these firms. While some empirical work on reverse factoring exists within research literature, our model constitutes the first analytic treatment of the problem, using the value framework of financial theory. We show how the value of reverse factoring results from, and is conditioned by (1) the spread in external financing costs, (2) the operating characteristics of the supplier, including the implied working capital policy, and (3) the risk-free interest rate. Thus, in addition to providing managerial insights that integrate operational and financial perspectives, our findings disclose an important relation of these elements to the broader macro-economic context.

Keywords: *Supply Chain Finance; Reverse Factoring; Joint Production and Financial Decisions; Capital Market Frictions.*

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1. Introduction

Since the seminal paper of Modigliani and Miller (1958), it is well known that financial and operational decisions are separable when capital markets are perfect and there are no frictions, e.g., taxes, transaction and financial distress costs, or information asymmetries. In this ideal case, the source of financing neither affects operating plans nor creates additional value. Capital markets in reality are not perfect, however, and firms face financial constraints and barriers when raising external funds. In particular, capital market frictions may create a significant wedge between the costs of internal and external funds for small- and medium-sized enterprises (SMEs). As the wedge gets larger, SMEs pay a greater external financing premium and are also more likely to under-invest and forgo positive net present value business opportunities (Hubbard, 1998; Carpenter and Petersen, 2002). This underinvestment can have negative repercussions further along a supply chain, since an SME may be a key supplier to a large corporation. We examine here a recent development that promises to address such problems: so-called “reverse factoring” arrangements, which facilitate information revelation and thus trigger affordable financing within a supply chain.

In a reverse factoring arrangement, a corporation (which we will frequently also denote by the term “OEM”) and its supplier (“SME”) work together with a bank, in order to optimize the financial flows resulting from trade. This optimization is possible when the corporation enjoys a lower short-term borrowing cost than its supplier, a fact that would typically be manifested by a relatively higher credit rating. Instead of communicating only with the supplier, the corporation registers approval of the latter’s invoices on an information system that is accessible to all three parties. The terms of the reverse factoring contract entail that, by registering approval on the system, the corporation guarantees that it will remit the corresponding amounts to the bank, after any agreed credit delay, for application to the supplier's account. If the supplier wishes to access these receivables earlier than the agreed delay, the bank will advance payment with a discount that is based on the corporation's borrowing rate, rather than the supplier's. This more favorable rate is possible because the transaction is fully and transparently collateralized by the payment guarantee of the corporation, i.e., the corporation pays the bank at the end of the trade credit period, irrespective of the financial condition of the supplier. Hence, reverse factoring serves as a mechanism mitigating the informational asymmetries regarding the supplier's assets and thereby enables cheaper financing.

Figure 1. Reverse Factoring Mechanism

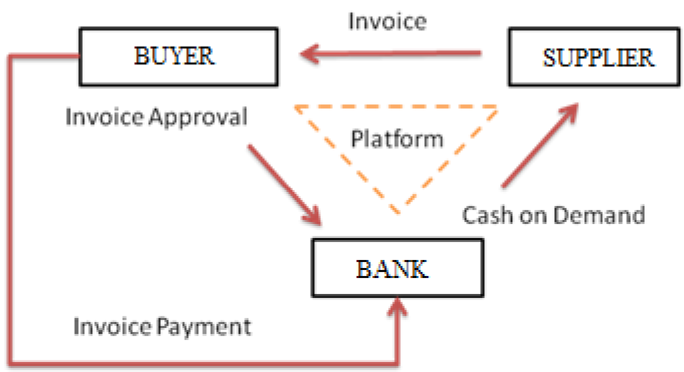


Figure 1 depicts the reverse factoring mechanism. It highlights liquidity improvement and/or lower short-term financing costs for the supplier, but further consideration shows that similar advantages for the corporation should be possible. Prior to using reverse factoring, the supplier's short-term financing expenses are a function of its own cost of external funds and the time that its receivables remain outstanding. If the reduction in the cost of external funds on transactions with the corporation is sufficiently large, the latter may extend the nominal trade credit period, while still leaving some net benefit for the supplier. A longer trade credit period entails that the corporation will also realize a reduction in short-term financing needs.

The impact of savings from reverse factoring may be profound. Rajan and Zingales (1998) note that accounts receivable and accounts payable represent, respectively and on average, 17.8% and 15% of firm value in the United States. Internationally, these figures range between 11.5% (average accounts payable in Germany) and 29.0% (average accounts receivable in Italy). Seifert and Seifert (2009) find that reverse factoring enables the corporations they study to realize average annual savings of $16 million on working capital financing. The aggregate savings for SMEs may be even greater, since the latter generally rely more heavily on short-term debt financing (Berger and Udell, 2002). This reliance can cause serious problems especially when credit is restricted. Shortage of working capital can lead to production delays, suboptimal stocking levels, higher prices, and even cases of financial distress among firms that are in principle economically sound. Small wonder that reverse factoring arrangements are "seen by many supply chain experts and managers as the great hope for easing problems with suppliers" (Milne 2009).

Nonetheless, in an uncertain business environment with operational decision making, it is not clear how reverse factoring creates value and how the terms of a reversed factoring arrangement should be specified, in order to guarantee that the corporation and bank may realize some benefit, without leaving nothing to the supplier. We provide a mathematical model for analyzing the operational and financial benefits of reversed factoring in a supply chain. To our knowledge, our work is the first to provide a rigorous analysis of this problem.

We consider two scenarios for stochastic demand. First, a single period make-to-order (MTO) model, where the raw materials purchases of the supplier can be delayed until the requirements of the corporation are revealed. Secondly, a make-to-stock model (MTS), where the supplier must make a stocking decision prior to the realization of demand. In both cases, the terms of any reverse factoring arrangement must be agreed at the outset. We assume that the supplier alternatively has access to conventional short-term financing, albeit at a less favorable rate. Our objective is thus to determine the conditions under which the reverse factoring contract will be economically viable and create value for each party.

Our work contributes to the operations management literature from three important angles.

- First, our models are cast in the value framework of finance: we discount for the riskiness of loans and exclude credit arbitrage. Here it becomes clear that the value of reverse factoring is determined by the difference in deadweight external financing costs faced by the corporation and supplier, not the nominal credit spread. Consequently, calculations based on nominal credit (which is common in practice) may significantly distort the magnitude of value calculations in reverse factoring contracts, generally to the disadvantage of the higher-risk supplier.
- Second, we quantify the value of reverse factoring contracts to the supply chain participants. We reveal the linkage between (1) the spread of deadweight external financing costs, (2) the operational characteristics of the supply chain (in particular, demand volatility) and the implied working capital policy of the supplier and (3) the risk-free interest rate. The interrelation of these factors offers a key insight to managers who are going to negotiate the terms of a reverse factoring contract. Not only a greater spread of deadweight external financing costs, but also a greater volatility in demand allows the corporation to increase its nominal payment period while retaining the participation of the supplier. We show, however, that the supplier's benefits are highly sensitive to this payment period extension: a small increase in payment delay may entail a relatively large

decrease in supplier's benefit. Payment period extension thus ultimately proves to be an inefficient strategy from a supply chain perspective, since it interacts with the operations of the supplier and reduces the total available benefits.

We find that suppliers with aggressive working capital polices will be more encouraged to use reverse factoring, and the total benefits for the supply chain are also larger in this case. The risk-free rate of interest does not influence the total benefits to the supply chain, but a lower risk-free rate gives the corporation less incentive to offer reverse factoring to the supplier. This latter finding yields interesting macroeconomic policy implications: in a credit squeeze (like the recent credit crisis of 2008), the spread in external financing costs between corporations and SMEs will tend to widen, augmenting the potential value of reverse factoring; yet lowering risk-free rates (a common monetary policy) may discourage the creation of supply chain finance solutions, further limiting smaller firms' access to capital.

- Third, in the MTS setting, we show that the reverse factoring may promote a significant increase in suppliers' stocking levels. This entails further value creation within the supply chain, in addition to any direct benefits that follow from reductions in external financing costs. Since the corporation shares in these operational benefits (by enjoying increased service levels), it is less inclined in this case to extend the nominal payment period compared to the MTO setting. We further observe that the corporation's benefits due to operational enhancement (operational benefits) may greatly exceed the direct financial benefits due to reduced external financing costs. For the supplier, under usual economic conditions, direct financial benefits dominate the operational benefits. We also show that operational enhancement and value creation does not necessarily imply each other when the risk free rate is positive.

Through a set of numerical analyses, we observe that reverse factoring, when implemented optimally and under reasonable economic conditions, may yield a supply chain value increase of 2 – 10%, due to enhanced operating and financing conditions.

2. Literature Review

The interface of operations and financial management is a burgeoning new area of research (Birge et al., 2007), and our work fits naturally with this literature. It is also closely related, however, to research streams that traditionally address the elements of our problem, such as small business finance and trade credit. Moreover, although we claim to provide the first analytic study of the reverse factoring mechanism, we note that other authors have presented important conceptual and empirical work on supply chain finance.

As mentioned in our introduction, the possibility that reverse factoring may offer some advantage to firms results from the failure of markets – or at least, the market for short-term commercial finance – to satisfy the requirements of the famous Modigliani-Miller theorems (Modigliani and Miller, 1958). Where capital markets are imperfect, the source of financing may impact other management decisions within the firm, and ultimately also the ability of firms to create value (Mayers and Smith, 1982; Smith and Stulz, 1985; Froot et al., 1993; Stulz, 1996). Irrespective of the cause, research on small business finance confirms that the financing costs faced by large corporations are significantly lower than what their SME suppliers can realize. For instance, Hennessy and Whited (2007) estimate that marginal equity flotation costs for large firms start at 5.0%, while the corresponding figure for small firms is 10.7%; bankruptcy costs amount respectively to 8.4% or 15.1% of capital. Gomes et al. (2003), in a general equilibrium asset pricing model, argue that bankruptcy costs of 5% can add 1.2% to the equity premium. As the bankruptcy costs (relative to the firm value) and other capital market frictions such as information asymmetries and transaction costs are usually more severe for SMEs (Cornell and Shapiro, 1988; Shane, 2003), these firms pay a larger deadweight external financing cost compared to their larger counterparts. As these frictions get more severe, access to capital by these small firms can be totally cut.

The existence of such differences is also a key motivation for firms' use of trade credit. Schwartz (1974) provides an early formalization of the financing advantage it provides. When sellers have better access to capital markets than their customers, trade credit allows them to offer better conditions than the customers could obtain through conventional financing. From a different angle, Smith (1987) suggests trade credit as a screening mechanism for suppliers to evaluate their customer's default risk. Many other finance and economic theorists have studied trade credit. See Petersen and Rajan (1997) for a detailed review. More recently, trade credit has been receiving attention from researchers working at the operations-finance interface. Gupta and Wang (2009) consider the impact of trade credit on supply chain contracting and inventory management and show that a base stock policy is still optimal under trade credit, provided the policy itself is adjusted to match the credit terms offered by the supplier. Lee and Rhee (2011) and Yang and Birge (2011) investigate trade credit as a tool for supply chain coordination and find that it serves as a risk sharing mechanism.

While some connection to trade credit is clear, the reverse factoring mechanism is different in several ways, as our analysis will show. A major contrast is evident in the orientation of the parties concerned: the credit provider in reverse factoring is situated downstream in the supply chain, while a typical trade credit arrangement involves a financially constrained small retailer that receives financing from a large supplier. More importantly, a reverse factoring transaction brings no additional risk for the corporation, since it is already obligated to pay the account receivable held by the supplier. In addition, when coupled with an extension of the payment period, reverse factoring entails that the corporation receives additional credit from its supplier, which to some degree will offset the benefit that the latter may realize through a cheaper borrowing rate.

Despite its apparent potential, the mechanism of reverse factoring has received relatively little attention within the research community. Studies that look specifically at reverse factoring restrict themselves to conveying high-level managerial assessments (e.g., Seifert and Seifert, 2009) or use financial ratios to calculate the benefits that should result for participants (e.g., Randall and Farris, 2009). We aim here to develop a rigorous framework by integrating financial and operational decision making to reveal the value creation mechanism of reverse factoring and understand how key market factors condition this value creation mechanism and distribution.

Turning finally to other recent research at the interface of operations and finance, we note frequent similarities with our concerns, even though no other work specifically addresses the reverse factoring problem. Babich and Sobel (2004) study the coordination of operational and financial decisions, in order to maximize the expected discounted proceeds from an initial public offering. Buzacott and Zhang (2004) link the financial capacity and operating decisions of a retailer by incorporating an asset-based constraint on the total amount of borrowing. In a newsvendor setting, they show that a lender should prefer to limit the size of a loan to the retailer, instead of charging a higher borrowing rate. Dada and Hu (2008) present a similar analysis, which formalizes the problem as a Stackelberg game and proves the uniqueness of the equilibrium. Protopappa-Sieke and Seifert (2010) consider the determination of optimal order quantity when a firm is subject to working capital restrictions. Our work is also related to Xu and Birge (2006), Hu et al. (2010) and Kouvelis and Zhao (2011).

3. The Base-Case Model: Make-to-Order with Deterministic Demand Size

In our base-case model of reverse factoring, we consider an SME that faces stochastic demand arrivals from a corporate buyer. The demand size is deterministic, and the SME operates in a make-to-order fashion. The SME is capital-constrained and may need to borrow in order to finance operations and meet current liabilities. At $t = 0$ the SME has cash reserves of magnitude y. Pre-existing current liabilities η, such as rents and rates, are due at end of the period, $t = \tau$. During the period, the firm may generate cash flow from operations, but the timing of cash receipts depends on the timing of demand from the OEM. For ease of exposition we assume that demand is realized at a single time-point and production is instantaneous. We denote the arrival time of demand by a random variable χ that has probability density function $\psi(.)$ and support $[0,\tau]$. Demand size ξ is constant and equal to μ in this base case (in subsequent sections, demand size will also be stochastic with expectation μ). Upon arrival of the demand, the SME purchases the required raw materials for cash, creates the final product, and sells it to the OEM on credit. Each unit of the final product is sold for price p and incurs a raw material cost c. The demand arrival thus instantaneously reduces the cash reserves of the SME by $c\xi$ and creates an account receivable (AR) of size $p\xi$. The nominal payment delay from the OEM to the SME is l_s time units, so the AR would normally be received after this delay. To avoid trivial cases, we assume that it is economical to use external funds to meet the demand. Our model also easily extends to the case where the SME is allowed a payment delay when purchasing raw materials.

The SME uses its cash reserves and cash generated from operations to meet the current liabilities as they come due. If this internal cash is not sufficient, external funds are received. In addition to any risk premium, the external finance provider charges deadweight costs to the SME, which we denote by β_s. That is, under the risk-neutral measure, external funds to the SME cost β_s per dollar above the risk-free rate, r_f. These deadweight costs may arise from the direct and indirect costs of bankruptcy and financial distress, as well as from informational asymmetries between managers and outside investors and transaction costs (Froot et al., 1993). Thus we denote the total external financing cost paid by the SME as $r_s = (r_f + \beta_s)$. By the same argument, external funds are available to the OEM at the cost $r_o = (r_f + \beta_c)$. (Note that these are not the nominal interest rates the parties face). Since established corporations are, in general, less exposed to capital market frictions, the deadweight costs for the OEM would be smaller than for the SME, i.e., we take $r_s > r_o > r_f$.

We assume that the SME's fixed assets are financed by a senior long-term debt that is due to be repaid at the end of the period. This senior debt includes covenants which may delay or otherwise restrict the repayment of any junior debts that come due before the end of the period. In particular, these restrictions could be imposed by the senior debtholders if the market value of the fixed assets fell below a specified threshold (*cf.* Leland 1994). While we do not explicitly model the value process for the SME's fixed assets, this underlying assumption drives the credit (default) risk of the bank loan and the allied risk premium that the SME must pay to obtain a junior loan. Hence, the credit risk of the SME is solely determined by the fixed assets and not influenced by the current business transaction with the corporate buyer (*cf.* Froot et al., 1993). Relaxing this assumption would only result in marginal insights while significantly limiting analytical tractability. For simplicity, we assume that OEM is an established corporation with negligible default risk. So, the deadweight cost of external funds for the OEM, β_c, is solely driven by transaction costs.

We assume that (1) the SME is economically viable and (2) only needs to borrow if the AR is not received before the fixed costs are due (this latter assumption will be relaxed when we discuss the stochastic demand size in Section 4). The OEM has sufficient internal financing and does not need external funds. The free cash reserves of both the SME and the OEM earn the risk-free rate. During the period, cash inflows and outflows to the SME take place in the following order.

1) <u>At $t = 0$.</u> SME starts operations with y amount of initial cash reserves (liquid assets).

2) <u>At $t = \chi$.</u> SME receives the order from the OEM and simultaneously places an order with its downstream supplier for the procurement of raw materials. Raw materials are purchased for cash and are received instantaneously. This reduces the SME's cash reserves to $ye^{r_f\chi} - c\xi$. For ease of exposition, we assume the SME starts operations with sufficient cash to make the raw material purchases. All other expenses associated with the order are due at the end of the period. The OEM instantaneously receives the final goods from the SME and sells them to the final customer(s) for w per unit.

3) <u>At $t = \tau$.</u> Short-term liabilities η of the SME come due. This event entails two possible cases.

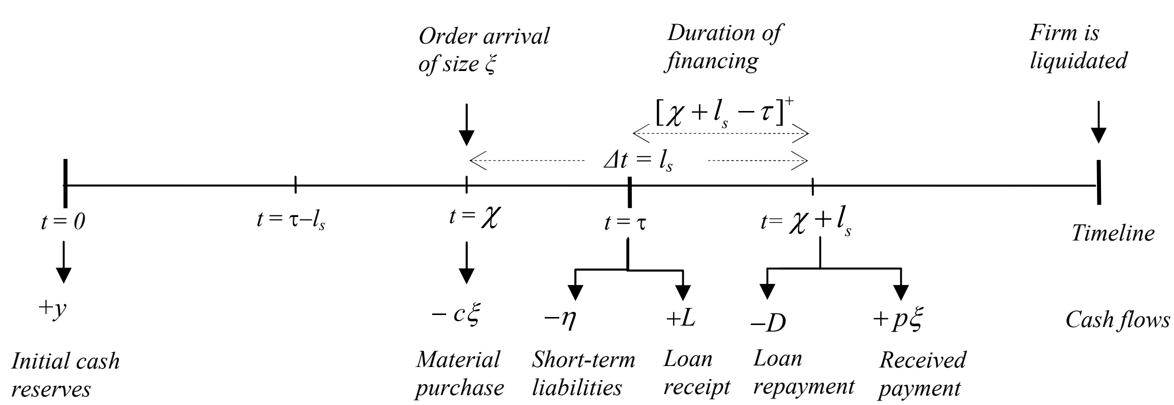
3.1. If $\chi + l_s \le \tau$ then the AR is received no later than the moment when the short-term liabilities are due, cash reserves at the end of period are $y_\tau = ye^{r_f\tau} - c\xi e^{r_f(\tau-\chi)} + p\xi e^{r_f(\tau-\chi-l_s)}$, and financing is not needed.

3.2. If $\chi + l_s > \tau$, then the AR are received after the short-term liabilities are due, the cash reserves at the end of period are $y_\tau = ye^{r_f\tau} - c\xi e^{r_f(\tau-\chi)}$, and this is insufficient to pay η. External funds in the amount of $L = \eta + c\xi e^{r_f(\tau-\chi)} - ye^{r_f\tau} > 0$ are received at a cost of r_s. These external funds are repaid at time $\chi + l_s$, i.e., when the SME receives the AR. Due to the credit risk involved in the debt, the uncertain loan repayment is denoted by D.

4) <u>At $t = \hat{\tau} \gg \tau$.</u> The SME closes its accounts and is liquidated.

The base-case model description and notation are summarized in Figure 2 below.

Figure 2. The MTO Model Description with Conventional External Financing



A reverse factoring arrangement entails three differences to the schema of cash flows presented in Figure 2. First, the SME may sell its approved invoice (accounts receivable) to the bank and receive a loan maturing at the nominal payment time of the OEM, which from the bank's perspective is assured by the creditworthiness of the OEM. Second, the OEM may extend the nominal payment period, and third, the bank may charge an additional premium for providing reverse factoring service, i.e., a transaction cost. Thus we characterize a reverse factoring contract by a vector of three parameters: (1) r_c, the reverse factoring financing cost for the SME, (2) l_c, the reverse factoring payment period of the OEM, and (3) *b*, the financier fee. The financier fee charged for the service is an additional deadweight cost for financing, i.e., $r_c = r_f + \beta_c + b = r_o + b$. The SME decides whether to accept the contract $v = (r_c, l_c, b)$.

Reverse factoring can generate value by reducing the deadweight financing costs of the SME. The savings are then shared between the SME and the OEM based on the contracting terms. All participants are assumed to maximize firm value. Furthermore, we assume that demand risk is diversifiable while the credit risk of the SME is driven by the changes in the value of the fixed assets which is correlated with the market and cannot be fully diversified. Hence, there exists a risk premium for loans issued to the SME. Under the risk-neutral measure, the value of the loan issued to the SME is

$$L = \frac{E^Q(D)}{e^{r_s \Delta l_s}}, \tag{1}$$

where $\Delta l_s = (\chi + l_s - \tau)$ gives the duration of the loan, and E^Q denotes expectation with respect to the risk neutral default probabilities of the SME. Note that r_s does not include the risk premium. The same set of assumptions also applies to the OEM and we take the deadweight costs of financing, i.e., the market frictions, to be constant over time.

In what follows, we use a first-order Taylor series approximation when discounting cash flows and use these values for the analytical models. Exact results only slightly differ due to the compounding of interest rates. The following propositions describe the benefits and the participation constraints of each party for a given reverse factoring contract. All proofs are included in the appendix.

PROPOSITION 1. *In an MTO business environment, the reverse factoring contract* $v = (r_c, l_c, b)$ *generates the following benefits for each party.*

(i) For the SME, $\pi^{sme} = \beta_s A(\tau - l_s)L(\mu) - (\beta_c + b)A(\tau - l_c)L(\mu) - p\mu r_f (l_c - l_s)$,

(ii) for the OEM, $\pi^{oem} = r_f p \mu (l_c - l_s)$,

(iii) for the entire supply chain, $\pi^{total} = \beta_s A(\tau - l_s)L(\mu) - (\beta_c + b)A(\tau - l_c)L(\mu)$,

Where $L(\mu) = \eta + c\mu - y$ *and* $A(\tau - l)$ *denotes the loss function, i.e.,* $E[\chi + l - \tau]^+$ *for* $l = l_s, l_c$.

In Proposition 1, functions *L*(.) and $A(.)$ denote respectively the SME's borrowing amount and the expected duration of borrowing. The SME's benefits are described by three terms: (1) $\beta_s A(\tau - l_s)L(\mu)$, the expected external financing cost of the SME without reverse factoring; (2)

$(\beta_c + b)A(\tau - l_c)L(\mu)$, the expected external financing cost with reverse factoring; and (3) $p\mu r_f(l_c - l_s)$, the opportunity cost of reverse factoring, resulting from any payment period extension $l_c - l_s$ imposed by the OEM. The SME can only benefit from the reverse factoring contract if the net expected cost of financing decreases under reverse factoring. The OEM collects additional risk-free earnings on the amount of AR for the duration of the payment period extension. (Recall that the OEM is assumed to have sufficient internal funds and does not need external financing.) Nonetheless, total benefits are independent of the risk-free rate and determined by the spread of external financing cost between the OEM and SME. In particular, when there are no market frictions, reverse factoring creates no value for the supply chain, which is in line with standard financial theory. We also note, however, that calculations based on the nominal spread in financing rates, which includes a risk premium, can distort these results, generally to the disadvantage of the more risky supplier.

The value of reverse factoring to the supply chain members depends on three key market factors: (1) the spread of deadweight cost of capital between the SME and the OEM; (2) the working capital policy and operational characteristics of the SME; and (3) the risk-free rate. As the credit spread between the parties widens, the SME experiences a larger decrease in its deadweight cost of financing and benefits more from reverse factoring. The working capital policy determines how often and how much external funding is needed by the SME. It is determined by the initial cash reserves, short-term liabilities and the operational characteristics such as the relative size and the volatility of SME's business. In particular, an aggressive working capital policy (i.e., lower cash reserves, more volatile business, higher short-term liabilities, etc.) implies more need for short-term financing and hence more benefits for the SME through reverse factoring, and vice versa. Finally, we see that opportunity cost to the SME is increasing (and benefits thus decreasing) with the risk-free rate, and this value is transferred directly to the OEM. The OEM's benefit from reverse factoring is driven by the expected business volume with the SME, the risk-free interest rate, and the payment period extension. The OEM's working capital savings are invested in short-term liquid assets and earn an additional risk-free interest rate for the duration of payment period extension. Table 1 below shows the sensitivity of the benefits described in Proposition 1 to key parameters.

	l_s	l_c	β_s	β_c	r_f	***Aggressiveness of WCP***
Supplier	Increases	Decreases	Increases	Decreases	Decreases	Increases
OEM	Decreases	Increases	Constant	Constant	Increases	Constant
Total	Increases	Decreases	Increases	Decreases	Constant	Increases

Table 1. Sensitivity of Reverse Factoring Benefits

The next proposition formalizes the participation constraint of each party and the value creation condition for the entire supply chain.

PROPOSITION 2. *In an MTO business environment, the following participation constraints apply to the reverse factoring contract* $v=(r_c,l_c,b)$:

(i) for the SME, $\beta_s A(\tau-l_s)L(\mu)-(\beta_c+b)A(\tau-l_c)L(\mu)\geq p\mu r_f(l_c-l_s)$,

(ii) for the OEM $l_c\geq l_s$,

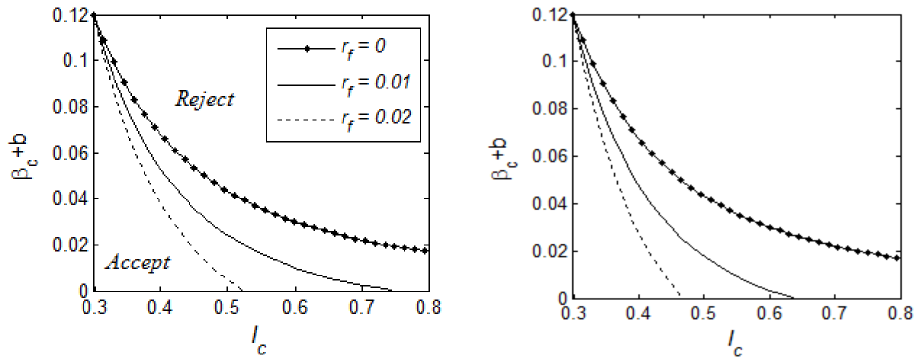
(iii) for the entire supply chain $\beta_s A(\tau-l_s)\geq(\beta_c+b)A(\tau-l_c)$.

The OEM requires a non-negative payment period extension while the SME's participation constraint is non-trivial, due to the interaction of payment period and financing cost with the operating characteristics and working capital policy. While the bank fee, *b*, linearly transfers wealth from the SME to the bank, the payment period extension has a more complex effect on the benefits of the SME. The total effect of the payment period extension is a compound of two distinct sub-effects. First, as the payment period increases, the SME bears the extra cost of financing over a longer time period. Second, with longer payment periods, it is more likely that the SME will deplete its internal cash and need external funds. Together, these sub-effects entail that the specific participation constraint of the SME, when viewed as the maximum financing rate allowable in the reversed factoring contract $v=(r_c,l_c,b)$, is not only a decreasing function of l_c, the payment period under reverse factoring, but also typically strictly convex. This relationship between l_c and β_c is further moderated by the magnitude of the risk-free rate: as r_f increases, the participation constraint becomes more limiting and the SME demands a larger reduction in its cost of external funds. Likewise, the participation constraint is more restrictive when the SME follows a conservative working capital policy, since in this scenario it uses external funds less often, while paying the same opportunity cost for reverse factoring.

Finally, it is important to note that when the risk-free rate is positive, the participation constraints of the supply chain and the SME diverge. In particular, a reverse factoring contract which generates value for the supply chain may be rejected by the SME if the opportunity cost of payment period extension is sufficiently large. Figure 3 below illustrates the preceding discussions of the participation constraint of the SME. For the numerical analyses we consider a base case with the following parameters: w = 15, p = 12, c = 10, r_f = 0.02, β_s = 0.12, b = 0.01, β_c = 0.01, μ = 100, l_s = 0.3, l_c = l_s+0.1, η = 1000, and y =1000. These parameters are used througout the remainder of the paper, unless otherwise mentioned. We assume that the demand arrival time is uniformly distributed over a period of one year, i.e., τ = 1. The results for other plausible demand arrival distributions, such as truncated normal, are qualitatively the same.

Figure 3. **SME's Participation Constraint as a Function of r_f, l_c and r_c**

(a) Under Aggressive WCP (y=1000) (b) Under Conservative WCP (y=1250)



Next, we describe the optimal reverse factoring contracts for each party and for the entire supply chain.

PROPOSITION 3. *In an MTO business environment, the reverse factoring contract* $v = (r_c, l_c, b)$ *maximizes*

(i) the SME's expected benefit when $(r_c = r_f + \beta_c + b, l_c = l_s, b = 0)$. *The maximum benefit for the SME is* $\tilde{\pi}^{sme} = (\beta_s - \beta_c) A(\tau - l_s) L(\mu)$.

(ii) The OEM's expected benefit when $(r_c = r_f + \beta_c + b, l_c = l_c^*, b = 0)$. *The maximum benefit for the OEM,* $\tilde{\pi}^{oem} = p\mu r_f (l_c^* - l_s)$, *where* l_c^* *is the payment period that makes the SME's participation constraint tight, i.e.,* $\beta_s A(\tau - l_s) L(\mu) - \beta_c A(\tau - l_c^*) L(\mu) = p\mu r_f (l_c^* - l_s)$.

(iii)The total supply chain benefit when $(r_c = r_f + \beta_c + b, l_c = l_s, b = 0)$. *The maximum supply chain benefit is* $\tilde{\pi}^{total} = (\beta_s - \beta_c) A(\tau - l_s) L(\mu)$.

A reverse factoring contract maximizes the SME's benefits when the payment period remains the same and the loan is issued at the borrowing rate of the OEM (i.e., when there are no bank fees for reverse factoring). The OEM breaks even under this contract and the SME receives all benefits that result from mitigating capital market frictions. This solution also maximizes the total supply chain benefits, which in this case are equal to the total expected savings in deadweight cost of financing. Since the OEM only benefits from an increase in the payment period, the OEM's maximum benefit occurs when the bank fee equals zero and the payment period is increased to the point where the SME breaks even. Nonetheless, Corollary 1 below shows that the OEM can at best extract only a fraction of the total supply chain benefit.

COROLLARY 1. *In an MTO business environment, increasing the payment period allows the OEM to extract at most only a fraction of the total supply chain benefit, i.e.,*

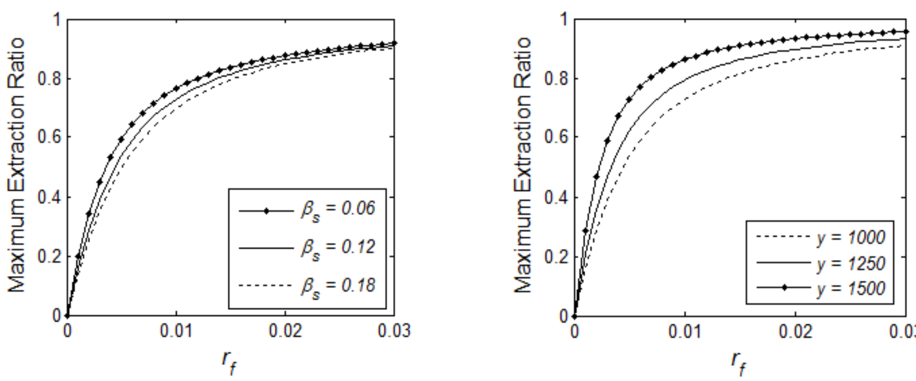
$$\frac{\hat{\pi}^{oem}}{\hat{\pi}^{total}} = \alpha = \frac{p\mu r_f (l_c^* - l_s)}{L(\mu)(\beta_s - \beta_c)A(\tau - l_s)} < 1, \tag{2}$$

where α is decreasing in β_s and η, increasing in β_c , r_f and y .

The OEM is unable to realize a maximum benefit equal to the maximum total supply chain benefit because of operational interactions that result when the payment period is extended. As discussed above, increasing the payment period not only imposes a larger financing period but also increases the chance that the SME will run short of cash due to the arrival of business opportunities during the period. This operational impact entails a non-linear reduction in the SME's benefits, limiting the possibilities for payment period extension. Contracts that do include an extension are thus inefficient, wasting some potential benefit of reverse factoring. The following figures show the OEM's maximum extraction ratio, α, as a function of the risk-free rate, the SME's original deadweight cost of financing, and the working capital policy of the SME.

Figure 4. OEM's Maximum Extraction Ratio and Absolute Value Generation

(a) Impact of r_f and β_s on α (b) Impact of r_f and WCP on α



Proposition 1 and Corollary 1 show that the maximum benefit fraction that the OEM can individually capture from the reverse factoring contract increases with the risk-free rate. As the credit spread increases, however, (note that β_c is fixed) the maximum benefits of the OEM grow at a slower rate than the total benefits. The OEM cannot efficiently extract these additional benefits by increasing the payment period due to the non-linear participation constraint of the SME. Hence α decreases (Figure 4(a)). The same reasoning explains the analogous effect of an aggressive working capital policy (Figure 4(b)).

4. Make-to-order with Stochastic Demand Size and Timing

In this section, we extend our base-case analysis by allowing demand size – as well as demand arrival time – to be stochastic. For the analytical derivations, we assume a general distribution for the demand size, ξ, with a probability density function $f(.)$, cumulative density function, $F(.)$, mean μ, and standard deviation σ. For the numerical results we adopt a normal distribution. For notational brevity, we use the variables defined in the previous section to denote their analogs here. The business model is still make-to-order, but the SME may now also need to seek financing upon demand realization, if internal cash is too little to procure all raw materials needed to meet the OEM's order. Further borrowing may also be needed at the end of the period, in order to make repayment of short-term liabilities. Nonetheless, we maintain the assumption that the discounted value of the SME's account receivable is always greater than any cash shortage that the SME experiences at the end of the period. This allows us to directly compare the reverse factoring and external financing options. (Allowing the firm to use external financing to cover the shortages in excess of the AR is possible but does not generate additional insights.) We find that the main results from the preceding analysis apply to the present case. The following proposition describes the benefit to each party.

PROPOSITION 4. *In an MTO business environment with stochastic demand size and timing, the reverse factoring contract* $v=(r_c,l_c,b)$ *generates the following benefits for each party:*

(i) For the SME, $\pi^{sme} = \beta_s L(l_s)-(\beta_c+b)L(l_c)-p\mu r_f(l_c-l_s)$,

(ii) for the OEM, $\pi^{oem} = p\mu r_f(l_c-l_s)$,

(iii) for the entire supply chain, $\pi^{total} = \beta_s L(l_s)-(\beta_c+b)L(l_c)$, *where,*

$$L(l)=E_{\xi,\chi}[(\eta+c\xi-y)\Delta l I_{\Omega_1^l}+\eta\Delta l I_{\Omega_2^l}+(c\xi-y)l I_{\Omega_3}],\ \Delta l=\chi+l-\tau,$$

$$\Omega_1^l=\{\xi,\chi:\xi\le y(1+r_f\chi)/c,\xi\ge(y(1+r_f\tau)-\eta)/(c(1+r_f(\tau-\chi))),\chi>\tau-l\},$$

$$\Omega_2^l=\{\xi,\chi:\xi\ge y(1+r_f\chi)/c,\chi>\tau-l\},$$

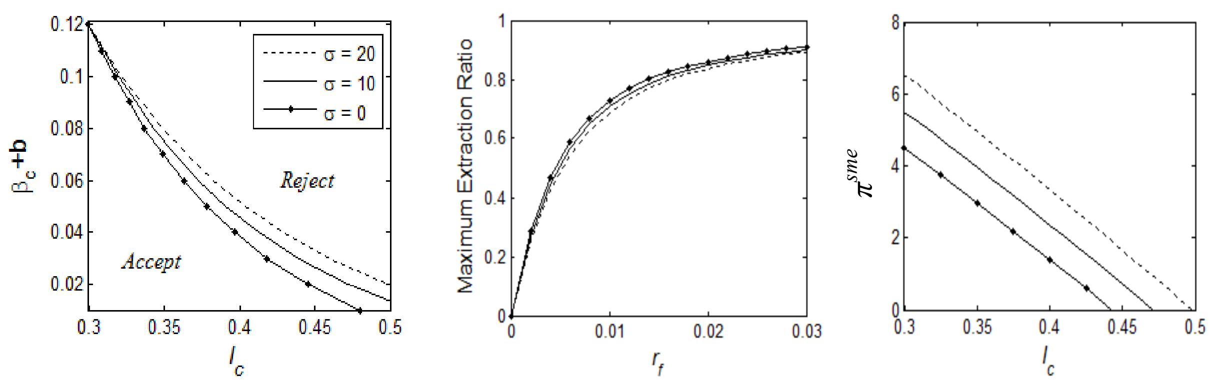
$$\Omega_3=\{\xi,\chi:\xi\ge y(1+r_f\chi)/c\},\ \ \textit{for } l=l_c \textit{ and } l_s.$$

Proposition 4 is analogous to Proposition 2, but the expected financing requirement has a different form, given by $L(.)$. In particular, with stochastic demand size, there are three possible cases where financing occurs. If $(\xi,\chi)\in\Omega_3$ then cash reserves are insufficient to procure all needed raw materials and the SME borrows to close the shortage when demand arrives. Note that since the production and sales are instantaneous, the resulting account receivable can be used to finance the purchase of raw materials (in practice this can be accomplished through a bridge loan which is repaid once the AR is created). If $(\xi,\chi)\in\Omega_2^l$ then, in addition to financing the purchase of raw materials, the SME needs to borrow to finance the payment of the short-term liabilities at the end of the period. If $(\xi,\chi)\in\Omega_1^l$ then the SME does not need to borrow to finance the purchase of raw materials, but needs to borrow to finance the payment of short-term liabilities.

The rest of the results from Section 3 hold here with slight modifications, but analytical tractability is limited. Proposition 3 and Corollary 1 still hold with financing needs $L(.)$ defined as in Proposition 4, and l_c^* is the positive value which makes the SME's participation constraint tight, i.e., $\beta_s L(l_s)-(\beta_c+b)L(l_c^*)=r_f p\mu(l_c^*-l_s)$. Next we provide a set of numerical analysis to illustrate the impact of demand size volatility on the results. For the numerical analysis, we use the same set of parameter values as in Section 3.

Figure 5. Effect of Demand Volatility

(a) on the SME's Participation Constraint (b) on α (c) on the Generated Value of the SME



Variability in demand size increases the need for external funds, so the benefit of reverse factoring for the SME increases with the variance of demand size (Figure 5(c)). This leads to an expansion of the SME's participation region, as shown by Figure 5(a). Higher demand variance also increases the OEM's maximum benefits, since the payment period can be extended further while retaining the SME's participation. Nonetheless, this exacerbates the inefficiencies resulting from payment period extension. The OEM's maximum benefits increase with variance, but the maximum extraction ratio of the OEM decreases as depicted in Figure 5(b), since the total benefits increase faster.

5. Make-to-Stock (MTS) Model

We now recast our model in a make-to-stock (MTS) setting, where the SME needs to produce and stock inventory prior to receiving demand from the OEM. We restrict the financing possibility to only the end of the period, i.e., financing will only be needed for the short-term liabilities (accordingly, we impose upper and lower limits on the demand size). The problem could of course be extended to allow for other possible financing needs, as in the model of Section 4. As noted in that case, however, the additional insights generated are marginal, while analytical tractability is greatly curtailed. The timeline of events and other assumptions for the MTS problem are as follows.

1) At $t = 0$. SME decides how much to produce and stock, Q, in order to meet the OEM's forthcoming demand. The SME purchases raw materials for cash. As in the MTO case, we assume that delivery and production lead times are zero. Accordingly, cash reserves immediately decrease to $y - cQ$. We assume the SME has sufficient cash to make the raw material purchases. Other short-term liabilities, η, are due at the end of the period.

2) At $t = \chi$. OEM observes demand ξ and places an order of same size with the SME. The SME immediately delivers the maximum quantity its inventory allows to meet the order, i.e., the SME sells $\min(\xi, Q)$. We assume that excess inventories have zero salvage value and excess demand is lost.

3) At $t = \tau$. Short-term liabilities η are due. This event entails two possible cases.

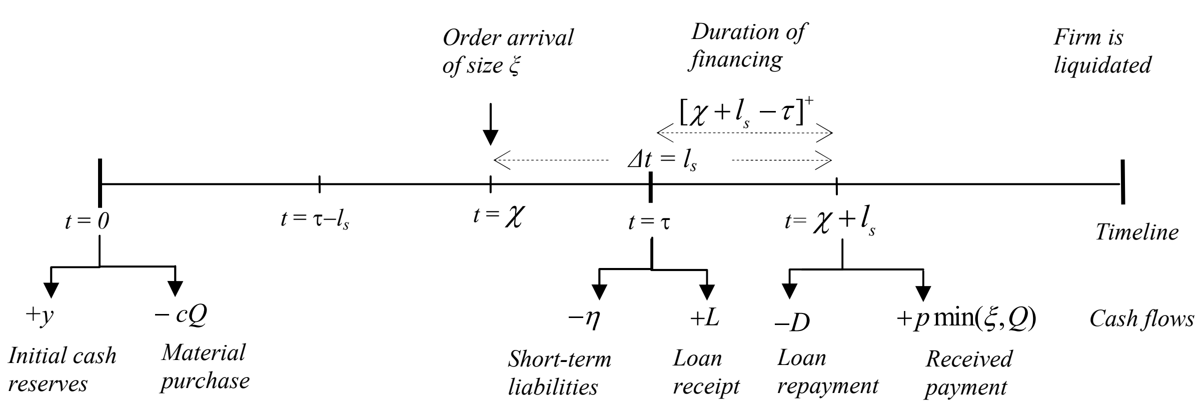
3.1. If $\chi + l_s \leq \tau$, the AR is received no later than the moment when short-term liabilities are due, cash reserves at the end of the period are $y_\tau = (y - cQ)e^{r_f \tau} + p\min(\xi, Q)e^{r_f(\tau-\chi)}$, and this is sufficient to pay η under the optimal policy and the worst-case demand scenario.

3.2. If $\chi + l_s > \tau$, the AR is not received before short-term liabilities are due. Cash reserves at the end of period are only $y_\tau = (y - cQ)e^{r_f \tau}$ and insufficient to pay η under the optimal policy. Hence the SME finances the shortage amount $\eta - (y - cQ)e^{r_f \tau}$ at the cost r_s. The financed funds are repaid upon maturity of the AR.

4) At $t = \hat{\tau} \gg \tau$. The SME closes its accounts and is liquidated.

The MTS model description with conventional external financing is depicted in Figure 6 below.

Figure 6. MTS Model Description with Conventional External Financing



The reverse factoring arrangement changes the terms of financing in the same way it does in the MTO case. The financing cost becomes r_c instead of r_s, and the OEM extends the nominal payment period from l_s to l_c. Here the outstanding AR is $pE\min(\xi, Q)$, and thus depends on the production quantity as well as the realization of demand. We first characterize the optimal quantity for the SME. This forms the basis for further analysis of value creation in this section.

PROPOSITION 5. *In an MTS business environment, the optimal production quantities of the SME, for the respective cases of conventional external financing and reverse factoring, are given by*

$$Q^*_{ext} = F^{-1}\left(\frac{p_{ext} - c_{ext}}{p_{ext}}\right),\ Q^*_{rev} = F^{-1}\left(\frac{p_{rev} - c_{rev}}{p_{rev}}\right).$$

The corresponding optimal values for the SME are

$$V^{sme}_{ext}(Q^*_{ext}) = pE\min(Q^*_{ext}, \xi)(1 - r_f(\bar{\chi} + l_s)) - cQ^*_{ext} - \beta_s L(Q^*_{ext})A(\tau - l_s),$$
$$V^{sme}_{rev}(Q^*_{rev}) = pE\min(Q^*_{rev}, \xi)(1 - r_f(\bar{\chi} + l_c)) - cQ^*_{rev} - (\beta_c + b)L(Q^*_{rev})A(\tau - l_c),$$

where $L(Q) = \eta + cQ - y > 0$, *for* $Q = Q^*_{ext}, Q^*_{rev}$ *and* $c_{ext} = c(1 + \beta_s A(\tau - l_s))$, $c_{rev} = c(1 + (\beta_c + b)A(\tau - l_c))$, $p_{ext} = p(1 - r_f(\bar{\chi} + l_s))$, $p_{rev} = p(1 - r_f(\bar{\chi} + l_c))$ *and* $\bar{\chi} = E[\chi]$.

Proposition 5 shows that the optimal production level of the SME is characterized by a newsvendor-type solution, modified by the deadweight external financing costs, payment period, and the risk-free rate. In particular, under costly external financing, the nominal purchasing cost of the SME increases by the expected external financing cost, $c_{ext} = c(1 + \beta_s A(\tau - l_s))$, and the sales price reduces by the expected value lost due to delay in the collection of revenues, $p_{ext} = p(1 - r_f(\bar{\chi} + l_s))$. With reverse factoring, the lower deadweight financing premium ($\beta_c + b \le \beta_s$) reduces the effective unit cost, while the payment period extension ($l_c \ge l_s$) non-linearly increases the financing needs and the unit cost, as well as linearly reducing the effective sales price in proportion to the risk-free rate. Accordingly, the operational enhancement due to reverse factoring depends on the net effect of these factors on the financing cost of the SME. Rearranging the critical ratios in Proposition 5, we observe that the SME produces more with reverse factoring if

$$\beta_s A(\tau - l_s)(1 - r_f(\bar{\chi} + l_c)) \ge (\beta_c + b)A(\tau - l_c)(1 - r_f(\bar{\chi} + l_s)) + r_f(l_c - l_s) \tag{3}$$

We refer to this constraint as the operational enhancement constraint (OEC). We next describe the benefits and the participation constraints (PC) of each party in the supply chain.

PROPOSITION 6. *In an MTS business environment, the reverse factoring contract* $v = (r_c, l_c, b)$ *generates the following benefits for each party.*

(i) For the SME,

$$\pi^{sme} = p_{rev} E\min(Q^*_{rev},\xi) - p_{ext} E\min(Q^*_{ext},\xi) - c(Q^*_{rev} - Q^*_{ext}) + \beta_s L(Q^*_{ext}) A(\tau - l_s) - (\beta_c + b) L(Q^*_{rev}) A(\tau - l_c),$$

(ii) for the OEM, $\pi^{oem} = E\min(Q^*_{rev},\xi)(\overline{w} - p_{rev}) - E\min(Q^*_{ext},\xi)(\overline{w} - p_{ext})$,

(iii) for the supply chain,

$$\pi^{total} = \overline{w}(E\min(Q^*_{rev},\xi) - E\min(Q^*_{ext},\xi)) - c(Q^*_{rev} - Q^*_{ext}) + \beta_s L(Q^*_{ext}) A(\tau - l_s) - (\beta_c + b) L(Q^*_{rev}) A(\tau - l_c),$$

where $\overline{w} = w(1 - r_f\,\overline{\chi})$, $L(Q) = \eta + cQ - y$ *for* Q^*_{ext}, Q^*_{rev} *and the corresponding participation constraints require that the expected benefits are non-negative.*

As in the MTO case, the payment period extension has a non-linear effect on the participation constraint of the SME. However, the effect is now more complex, due to its interaction with the production decision. The OEM's participation constraint also trades off financial and operational benefits as the payment period varies. Note that the participation constraint of the SME does not necessarily coincide with the quantity enhancement condition described above, unless the risk-free rate is zero. In the following we describe the relationship between the participation and quantity enhancement constraints of the SME.

COROLLARY 3. *In an MTS business environment,*

(i) when $r_f = 0$*, the SME's participation and operation enhancement constraints are identical and given by*

$$\beta_s A(\tau - l_s) \geq (\beta_c + b) A(\tau - l_c),$$

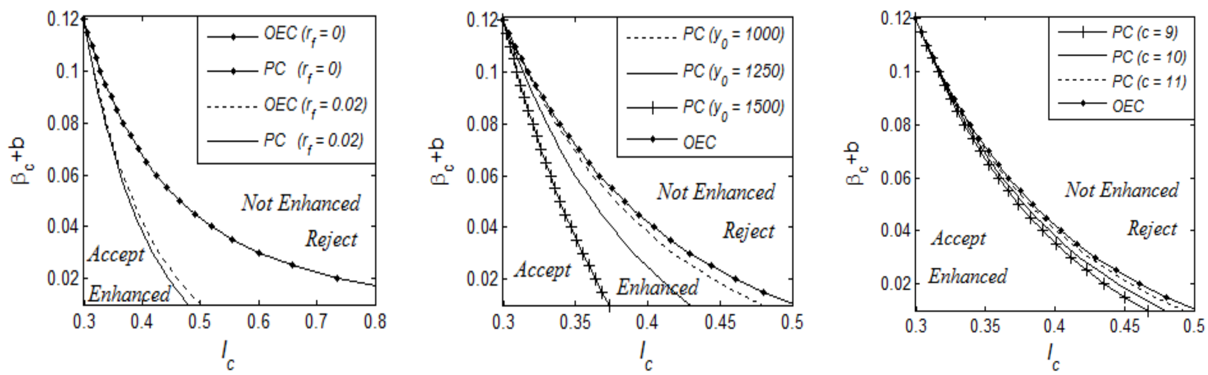
(ii) when $r_f > 0$*, there exists a threshold* $\hat{y}$ *such that if* $y > (<) \hat{y}$ *then the SME's participation constraint is tighter (looser) than the operational enhancement constraint.*

When the risk-free rate is zero, there is no opportunity cost for reverse factoring, and hence $p_{ext} = p_{rev} = p$. In this case, a quantity enhancement implies a reduction in the net unit cost, i.e., $c_{rev} < c_{ext}$, and hence also implies value creation. When the risk-free rate is positive, however, reverse factoring involves an opportunity cost depending on the risk-free rate and the payment period extension. In particular, this effect translates into a reduction in the effective sales price $p_{rev} < p_{ext}$. So, in this case, a quantity enhancement does not necessarily improve the value, since the reduction in the sales price may dominate benefits arising from the reduction in the unit cost (even when the critical ratio increases). Figure 7(a) provides a numerical illustration of this phenomenon. For the numerical analysis, we use the same base case as in Section 3 together with $\sigma = 20$.

As the SME's working capital policy becomes more conservative (i.e., y increases), the divergence of participation and operational enhancement constraints becomes more emphasized (Figures 7(b)). If the SME has a very conservative working policy (for example, when the initial cash reserves are well above the fixed payments), then very little financing is needed but a relatively large opportunity cost is incurred. In these cases, this opportunity cost is more likely to dominate the benefits of operational enhancement. Indeed, Figure 7(b) shows that when $y > \hat{y}$, as the working capital policy of the SME becomes more conservative (i.e., as y increases), the wedge between OEC and PC widens. In this case, even if operations are enhanced, the SME would rather reject a reverse factoring contract.

Figure 7. Comparison of Participation (PC) and Operation Enhancement (OEC) Constraints

(a) Effect of r_f (b) Effect of y (c) Effect of c



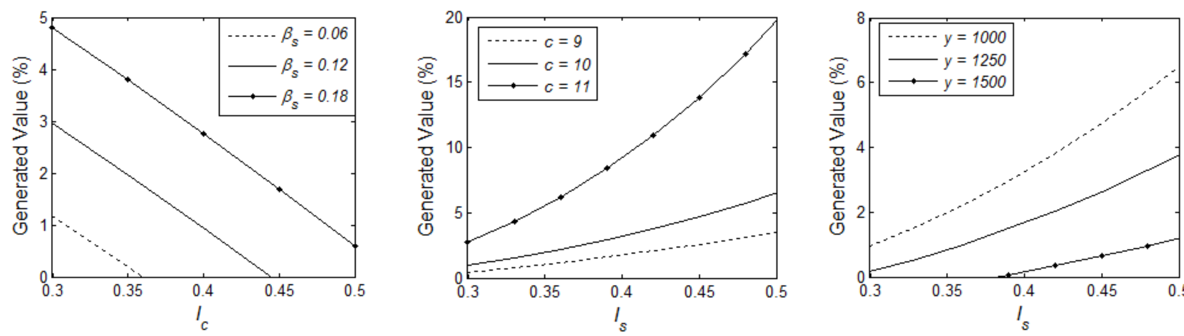
On the other hand, Corollary 3 also shows that as the SME needs greater financing (i.e., when $y < \hat{y}$) the participation constraint becomes tighter than the operational enhancement constraint. That is, the SME chooses to participate, even if production quantity decreases due to reverse factoring. In these cases, SME's opportunity cost for reverse factoring is relatively low since in expectation he borrows a larger amount. So, financial savings may dominate a possible reduction in value due to reduced operating level. In Figure 7(c), we illustrate effect of profit margin (or equivalently the effect of a change in the unit cost, c). Increasing the profit margin (lowering c) provides the firm with more internal cash flows to meet the cash outflows, and hence results in a similar effect to increasing initial cash reserves y.

In what follows, we numerically explore the value generated by reverse factoring for the SME and OEM as described in Proposition 6 above. We define the percentage value generation for the SME as $100*[V_{rev}^{sme}(Q_{rev}^*) - V_{ext}^{sme}(Q_{ext}^*)]/V_{ext}^{sme}(Q_{ext}^*)$. This is analogously defined for the OEM. Figures 8(a)-(c) below show that reverse factoring may lead to significant value creation for the SME. In particular, reverse factoring offers more value generation for low profit margin SMEs (high c) with aggressive working capital polices (low y) that operate in business environments with long initial payment periods (high l_s) and pay a high cost for external funds (high β_s). In such cases, the SME on average needs to borrow a larger sum of money for a longer period of time and pays a higher cost for these funds. In Northern European countries, payment periods are typically less than 90 days, so value generation from reverse factoring will be lower. In China and Southern European countries such as Spain and Italy, payment periods approach (and even exceed) six months (Euler Hermes, 2012), so value generation will be much greater there, provided contracts are implemented properly. Thus it is no surprise that Santander Group, a

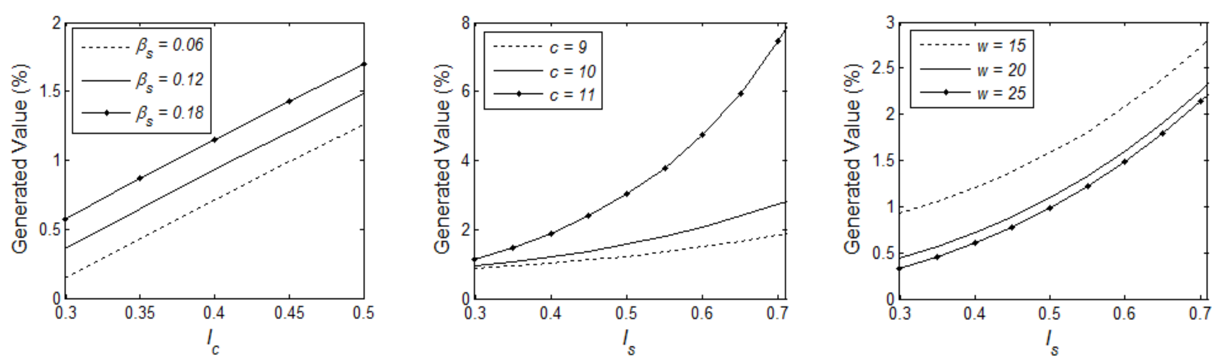
Spanish bank, is a leading provider of reverse factoring services. Nonetheless, it is also important to note that the extended payment period l_c quickly reduces the total value generation, as we saw in the MTO case. Insights on value generation for the OEM (Figures 8(d)-8(f)) are similar. Under the MTS model, in addition to the extended payment period, l_c, the OEM may also significantly benefit from quantity enhancement at the SME. In particular, as β_s, c, and l_s go up, the OEM shares a significant portion of the quantity enhancement benefits due to reverse factoring and hence the OEM's benefits quickly go up. As expected this effect is more emphasized when the OEM's margin is low (i.e., w is low).

Figure 8. Percentage Value Generation for the SME and the OEM

(a) Impact of β_s, and l_c on the SME (b) Impact of l_s and c on the SME (c) Impact of l_s and y on the SME



(d) Impact of β_s, and l_c on the OEM (e) Impact of l_s and c on the OEM (f) Impact of l_s and w on the OEM



We now explore the identified benefits in greater detail. Under the MTS operating model, value of reverse factoring stems both from reduced external financing costs as well as enhanced operating levels at the SME. First, suppose the SME does not change its operating plan under the reverse factoring scheme and produces the amount Q^*_{ext}. The entire benefits of the SME are then due to the reduction in external financing costs. We define the quantity $F^{sme} = V^{sme}_{rev}(Q^*_{ext}) - V^{sme}_{ext}(Q^*_{ext})$ to be the financial benefits of the SME:

$$F^{sme} = -r_f p(l_c - l_s)E\min(Q^*_{ext}, \xi) + L(Q^*_{ext})\big(\beta_s A(\tau - l_s) - (\beta_c + b)A(\tau - l_c)\big). \tag{4}$$

Any other benefits to the SME in excess of F^{sme} come from enhancements in operations. We define these as $O^{sme} = [V_{rev}^{sme}(Q_{rev}^*) - V_{ext}^{sme}(Q_{ext}^*)] - F^{sme}$, where the term in brackets is the total benefits from Proposition 6(*i*). This leads to

$$O^{sme} = p_{rev}\left(E\min(Q_{rev}^*,\xi) - E\min(Q_{ext}^*,\xi)\right) - c_{rev}(Q_{rev}^* - Q_{ext}^*). \tag{5}$$

The financial benefits are very similar to the benefits in the MTO case, which makes intuitive sense. Once the production level is fixed, the SME saves the expected differential in deadweight financing costs, and pays the opportunity cost that results from any payment period extension by the OEM. Benefits or losses accrued beyond this amount are driven by the changes in the production level.

We can also distinguish the financial and operational benefits of reverse factoring for the OEM:

$$F^{oem} = r_f p(l_c - l_s)E\min(Q_{ext}^*,\xi), \tag{6}$$

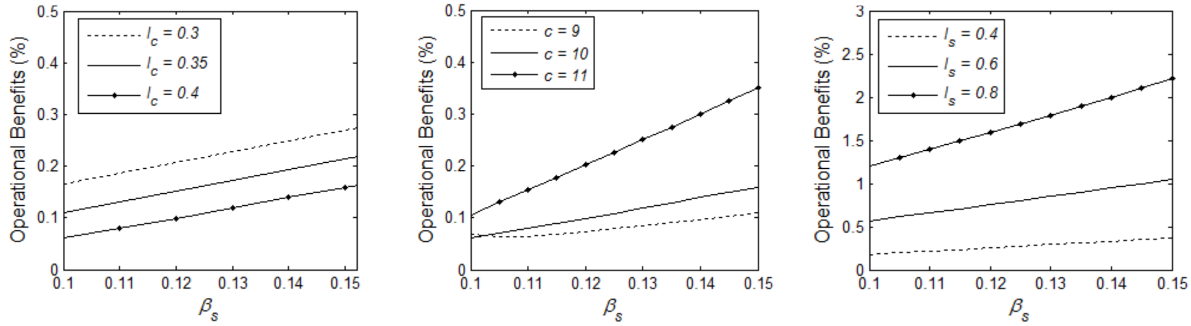
$$O^{oem} = (\bar{w} - p_{rev})(E\min(Q_{rev}^*,\xi) - E\min(Q_{ext}^*,\xi)). \tag{7}$$

When the production level is fixed, the OEM earns the risk-free rate on the expected business volume for the duration of payment extension. Remaining benefits to the OEM follow from quantity enhancement at the SME level, which is described by $(\bar{w} - p_{rev})(E\min(Q_{rev}^*,\xi) - E\min(Q_{ext}^*,\xi))$. Intuitively, when the OEM's margin is large (i.e., $\bar{w} - p_{rev}$ is high) and the risk-free rate is low, operational benefits in (7) are likely to dominate the financial benefits in (6).

The total benefits for the SME in Figure 8 are mostly due to savings in external financing costs. Figure 9 shows the relative contribution of operational benefits to the total value generation (i.e., $100 * O^{sme}/(F^{sme} + O^{sme})$), which is small for our base case. Intuitively, if the SME's net operating margin is low, then the external financing costs have a significant effect on the production volume and reverse factoring would entail a significant increase in production amount. However, this change in the operating level would have a relatively small effect on the net profits since the operating margin is thin, and financial savings due to reduced external financing costs would have a much bigger effect. On the other hand, when the net operating margin of the SME is high it needs less external financing and the cost of external funds has a relatively small effect on the quantity decision. Indeed, from Figure 9 we observe that operational benefits are dominated by the financial benefits for the SME. Although not presented in Figure 9, we have numerically observed that the operational benefits may exceed 50% if the payment period is very high (l_s > 0.8) and the SME profitability is very low (c>11).

Figure 9. The Relative Size of Operational Benefits Compared to the Total Benefits for the SME

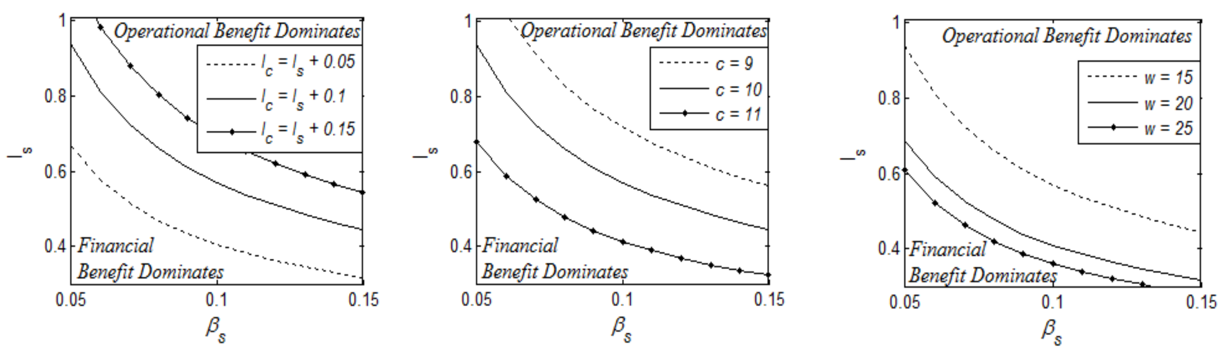
(a) Impact of l_c (b) Impact of c (c) Impact of l_s



For the OEM, however, operational benefits may easily dominate the financial benefits. In particular, quantity enhancement at the SME immediately translates into higher service levels for the OEM, and when the OEM's profit margin is large this may create benefits well in excess of the financial benefits arising from extended payment period. Figure 10 shows the parameter spaces where the operational benefits due to quantity enhancement account for more than 50% of total value generation for the OEM. We observe that the operational benefits tend to dominate the financial benefits with larger credit spread, longer nominal payment period, greater OEM profitability, smaller SME profitability and shorter payment period extension. In general, under these cases, there will be a larger quantity enhancement at the SME.

Figure 10. Conditions under Which the Operational Benefits Dominate the Financial Benefits of the OEM

(a) Impact of l_c (b) Impact of c (c) Impact of w



The next proposition discusses the individually optimal reverse factoring contracts for each party.

PROPOSITION 7. *In an MTS business environment, a reverse factoring contract* $v = (r_c, l_c, b)$ *maximizes*

(i) the SME's expected benefits when $(r_c = r_f + \beta_c, l_c = l_s, b = 0)$,

(ii) the OEM's expected benefit when $(r_c = r_f + \beta_c, l_c = l_c^*, b = 0)$, *where*

$$l_c^* = \underset{l_s \le l_c \le l_c'}{\arg\max} \{E \min(Q^*_{rev}, \xi)(\bar{w} - p_{rev}) - E \min(Q^*_{ext}, \xi)(\bar{w} - p_{ext})\},$$

and l_c' *solves*

$$p_{rev} E \min(Q^*_{rev}, \xi) - p_{ext} E \min(Q^*_{ext}, \xi) - c(Q^*_{rev} - Q^*_{ext}) + \beta_s L(Q^*_{ext}) A(\tau - l_s)$$
$$-(\beta_c + b) L(Q^*_{rev}) A(\tau - l_c) = 0$$

(iii) The total supply chain benefit when $(r_c = r_f + \beta_c, l_c = l_s, b = 0)$.

As in the MTO case, the SME benefit and total supply chain benefit are maximized when the payment period is not extended and there is no transaction cost for reverse factoring. However, for the OEM an interior solution can be optimal in the MTS case. Due to the internalized operational benefits, the OEM may prefer not to make the SME's participation constraint binding, but may rather sacrifice some direct financial benefits in return for higher stocking levels at the SME. This depends to a large extent on the profit margin of the OEM. If the OEM's profit margin is relatively large, then the marginal value of increasing the payment period is low and may even become negative before the SME participation becomes tight. When the OEM's profit margin is relatively low, it is more likely to collect all the financial benefits by making the SME's participation tight. (In order to focus on the effect of reverse factoring we excluded the possibility of reducing payment period below l_s, the original level.)

6. Discussion

This research reveals the interaction of operational and financial decisions in a supply chain where financing is facilitated by a reverse factoring arrangement. We disclose the value creation mechanism of reverse factoring and explain how this value is conditioned by key market factors. We consider make-to-order and make-to-stock models and explore the impact of reverse factoring on the operating decisions and the value implications of the supply chain members. In Section 6.1 below we discuss the managerial implications of our findings, then we offer conclusions and acknowledge limitations in Section 6.2.

6.1 Managerial Implications

Our analysis of reverse factoring contracts links their potential for value creation to (1) the spread of external financing costs between the OEM and the SME, (2) the operational characteristics of the SME (in particular, demand volatility and the implied working capital policy) and (3) the risk-free interest rate. The interrelation of these factors constitutes a key insight for managers who will negotiate the terms of a reverse factoring contract.

In an MTO environment, managers should assess their deadweight external financing cost, working capital policy and the risk-free interest rate to value reverse factoring agreements. Reverse factoring is most beneficial to the supply chain members when the spread in deadweight financing costs is high, nominal payment periods are long, the demand volatility is high, and the SME employs an aggressive working capital policy. The risk-free interest rate does not impact the total value generation, but it significantly influences the individual benefits of the OEM and SME. A low risk-free interest rate may even create a disincentive to the establishment of reverse factoring contracts, since the benefits to the OEM may be reduced below a critical threshold.

Managers should be cautious about the effects of payment period extension on the SME's operations and potential value creation. A small increase in payment delay may entail a relatively large decrease in SME's benefit from reverse factoring. Payment period extension thus ultimately proves to be an inefficient strategy, since it interacts with the operations of the SME and reduces the total benefits available to the supply chain. While SME managers must clearly be careful not to agree to a value-destroying contract, OEM managers should also be cautious not to hurt their SME suppliers through a poorly-arranged reverse factoring contract. Contracts with long payment period extensions may benefit the OEM in the short term, but may in the long term adversely affect the financing of the SME and endanger the supply lines of the OEM.

In an MTS environment, reverse factoring contracts should be evaluated with consideration for the operational implications as well as the financial implications. In an MTO environment, the benefits of reverse factoring are solely driven by savings in expected deadweight financing costs. Contrastingly, reverse factoring in an MTS setting also generates benefit through operational enhancement, besides any saving in financing costs. For the OEM the size of the operational benefits may easily exceed the financial benefits when the SME's profit margin is low, the OEM's margin is high, the credit spread is high, and the nominal payment periods are long. The situation is different for the SME, whose financial benefits typically dominate any operational benefits. Nevertheless, in an MTS business model the OEM is less inclined to extend the nominal payment period, since it shares in the operational benefits by enjoying increased service levels.

When the risk-free interest rate is not zero, SME managers should be aware that operational enhancement due to reverse factoring does not imply value creation and vice versa. Our analysis draws attention to the importance of risk-free interest rate for the design of reverse factoring contracts. We show that when the risk-free rate is positive, operational enhancement and value creation do not imply each other for the SME. The opportunity cost of reverse factoring due to payment period extension should be carefully evaluated by the SME managers. In particular, the managers may participate in reverse factoring even when the production level decreases or may choose not to participate even when the production level increases.

6.2 Conclusions

The recent credit crisis emphasized the need for and the value of close financial and operational collaboration among the supply chain members. In this paper, we explore a particular form of intra-chain collaboration, reverse factoring, which many businesses see not only as an important element in their strategy for recovery from the recent credit crisis, but also as a means for generating more value during usual economic conditions. Reverse factoring entails that supply chain partners collaborate on financing arrangements, in order to improve the efficiency of their transactions, reduce costs, and obviate operational roadblocks.

We consider a single period stylized model of trade between a supplier and a corporate customer. Our analysis can be extended to a multi-period setup with parties continuously engaging in trade over time. Numerically, we have observed that a multi-period model qualitatively results in similar managerial insights while the analytical tractability is significantly limited. Our analysis also suggests several related directions for exploration. For instance, it would be also interesting to explore reverse factoring in an international setting in the presence of exchange and interest rate risks. Integrating financial risk management and reverse factoring contracting can be a fruitful research direction. Furthermore, an exploration of the dynamics and the value creation mechanism of pre-shipment financing, where there is performance risk in terms of the delivery of the final output, would be a useful complement to the post-shipment financing model presented here.

Our analysis should enable the supply chain managers to design more efficient reverse factoring contracts and further optimize their operational and financial benefits. The results provide managers with guidance that could facilitate access to working capital and increase the financial efficiency of the supply chain. This leads to cost savings, which in return yield further profits for the participants and help ensure the survival of the business relations.

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Appendix 1: Proofs

Proof of Proposition 1.(i)

Let us first consider the cash flows to the SME under the conventional external financing case. Note that the free cash reserves continuously earn the risk-free rate. By assumption, if the demand arrives before $t=\tau-l_s$, then the SME does not need to borrow, otherwise the firm borrows, $L_{t=\tau}=\eta+(c\mu-ye^{r_f\chi})e^{r_f(\tau-\chi)}$, at time τ and pays back a stochastic amount D (due to credit risk) at time $t=\chi+l_s>\tau$. Under the risk-neutral measure, the loan is priced by the bank so that:

$$L_{t=\tau}=\frac{E^Q(D)}{e^{r_s\Delta l_s}},$$

where $\Delta l_s=(\chi+l_s-\tau)$ is the loan duration. Hence for a given χ, from Figure 2, cash flows discounted to t = 0, is given by:

$$\begin{aligned}
V_{ext}^{sme}(.\,|\,\chi) &= \frac{p\mu}{e^{r_f(\chi+l_s)}}-\frac{c\mu}{e^{r_f\chi}}+\left(\frac{L_{t=\tau}}{e^{r_f\tau}}-\frac{E^Q[D]}{e^{r_f(\chi+l_s)}}\right)I_{\{\chi>\tau-l_s\}} \\
&= \frac{p\mu}{e^{r_f(\chi+l_s)}}-\frac{c\mu}{e^{r_f\chi}}+\left(\frac{L_{t=\tau}}{e^{r_f\tau}}-\frac{e^{r_s\Delta l_s}L_{t=\tau}}{e^{r_f(\chi+l_s)}}\right)I_{\{\chi>\tau-l_s\}} \qquad \left(L_{t=\tau}=\frac{E^Q(D)}{e^{r_s\Delta l_s}}\right) \\
&= \frac{p\mu}{e^{r_f(\chi+l_s)}}-\frac{c\mu}{e^{r_f\chi}}+(1-e^{\beta_s\Delta l_s})L_{t=0}I_{\{\chi>\tau-l_s\}} \qquad \left(L_{t=0}=\frac{L_{t=\tau}}{e^{r_f\tau}},\ \Delta l_s=\chi+l_s-\tau\right)
\end{aligned}$$

Applying the first order Taylor series approximation with respect to (r_f, β_s) around (0, 0) we obtain:

$$\hat{V}_{ext}^{sme}(.\,|\,\chi)=p\mu(1-r_f(\chi+l_s))-c\mu(1-r_f\chi)-\beta_s\Delta l_s(\eta+c\mu-y)I_{\{\chi>\tau-l_s\}}$$

Then, the value of the cash flows at t = 0 is

$$\hat{V}_{ext}^{sme} = E_\chi \hat{V}(.\,|\,\chi) = E_\chi[p\mu(1-r_f(\chi+l_s)) - c\mu(1-r_f\chi) - \beta_s \Delta l_s(\eta + c\mu - y)I_{\{\chi>\tau-l_s\}}],$$
$$= p\mu(1-r_f(\overline{\chi}+l_s)) - c\mu(1-r_f\overline{\chi}) - \beta_s L(\mu)A(\tau\text{ - }l_s).$$

where $\overline{\chi} = E(\chi)$, $L(\mu) = (\eta + c\mu - y)$ and $A(\tau\text{ - }l) = E_\chi[\Delta l I_{\{\chi>\tau-l\}}]$ for $l=l_s$, l_c.

Similarly, the value of the cash flows under reverse factoring is

$$\hat{V}_{rev}^{sme} = p\mu(1-r_f(\overline{\chi}+l_c)) - c\mu(1-r_f\overline{\chi}) - (\beta_c + b)L(\mu)A(\tau\text{ - }l_c).$$

Then, the expected benefits of the SME from reverse factoring are determined by:

$$\pi^{sme} = \hat{V}_{rev}^{sme} - \hat{V}_{ext}^{sme} = \beta_s A(\tau\text{ - }l_s)L(\mu) - (\beta_c + b)A(\tau\text{ - }l_c)L(\mu) - p\mu r_f(l_c - l_s).$$

Note that for notational simplicity we present $\hat{V}_{rev}^{sme}$ and analogous terms as V_{rev}^{sme} in the main text.

□

Proof of Proposition 1.(ii)

Under conventional external financing, the value of the cash flows to the OEM, for a given χ, at $t = 0$ is given by (note that there is no default risk for the OEM):

$$V_{ext}^{oem}(.\,|\,\chi) = -\frac{p\xi}{e^{r_f(\chi+l_s)}}$$

Applying the first order Taylor series approximation with respect to r_f around 0, we get:

$$\hat{V}_{ext}^{oem} = -E_\chi[p\mu(1-r_f(\chi+l_s))] = -p\mu(1-r_f(\overline{\chi}+l_s))$$

Similarly, the value of the cash flows under reverse factoring is:

$$\hat{V}_{rev}^{oem} = -p\mu(1-r_f(\overline{\chi}+l_c))$$

Accordingly, the benefits of the OEM from reverse factoring are determined by:

$$\pi^{oem} = \hat{V}_{rev}^{oem} - \hat{V}_{ext}^{oem} = p\mu r_f(l_c - l_s).$$

□

Proof of Proposition 1.(iii)

Adding up the OEM's and SME's benefits in parts (*i*) and (*ii*) gives the desired result.

□

Proof of Proposition 2

Directly follows from the proof of Proposition 1.

□

Proof of Proposition 3.(i)

The optimal contract for the SME is obtained by maximizing the SME's benefits subject to the participation constraint of the OEM and non-negative bank fees:

$$\hat{\pi}^{sme} = \max_{b,l_c} \pi^{sme} = \max_{b,l_c} L(\mu)\big(\beta_s A(\tau - l_s) - (\beta_c + b)A(\tau - l_c)\big) - p\mu r_f (l_c - l_s)$$
$$s.t.\ l_c \ge l_s, b \ge 0.$$

Observing that the first partials of the objective function are non-positive proves the desired result:

$$\frac{\partial \pi^{sme}}{\partial b} = -\beta_c L(\mu) A(\tau - l_c) < 0.$$
$$\frac{\partial \pi^{sme}}{\partial l_c} = -(\beta_c + b)L(\mu)\frac{\partial A(\tau - l_c)}{\partial l_c} - p\mu r_f < 0 \qquad (\frac{\partial A(\tau - l_c)}{\partial l_c} > 0)$$

□

Proof of Proposition 3.(ii)

The optimal contract for the OEM is obtained by maximizing the OEM's benefits with respect to the contract terms subject to the participation constraint of the SME and non-negative bank fees:

$$\hat{\pi}^{oem} = \max_{b,l_c} \pi^{oem} = \max_{b,l_c} p\mu r_f (l_c - l_s)$$
$$s.t.\ L(\mu)\beta_s A(\tau - l_s) \ge L(\mu)(\beta_c + b)A(\tau - l_c) + p\mu r_f (l_c - l_s)$$
$$b \ge 0.$$

It is obvious that the objective function is increasing in l_c and constant in bank fees b. Now observe that the right hand side of the SME's participation constraint is monotone increasing in l_c and b. This implies that optimal b is zero, and the optimal l_c makes the SME participation tight.

□

Proof of Proposition 3.(iii)

The optimal contract for the supply chain is obtained by maximizing the total supply chain benefits with respect to the contract terms subject to the participation constraints of the SME and OEM, and non-negative bank fees:

$$\hat{\pi}^{total} = \max_{b,l_c} \pi^{total} = \max_{b,l_c} L(\mu)\big(\beta_s A(\tau - l_s) - (\beta_c + b)A(\tau - l_c)\big)$$
$$s.t.\ L(\mu)\big(\beta_s A(\tau - l_s) - (\beta_c + b)A(\tau - l_c)\big) \ge r_f p\mu(l_c - l_s),$$
$$l_c \ge l_s, b \ge 0.$$

First, observe that the objective function is decreasing in b and l_c, and the right hand side of the SME's participation constraint is monotone increasing in b and l_c. Together with the constraints $l_c \ge l_s$ and $b \ge 0$, this implies the desired result.

□

Proof of Corollary 1

Recall that the OEM`s maximum benefit is given by $r_f p\mu(l_c^* - l_s) = L(\mu)\big(\beta_s A(\tau - l_s) - \beta_c A(\tau - l_c^*)\big)$.

Then,

$$\frac{\widehat{\pi}^{oem}}{\widehat{\pi}^{total}} = \frac{r_f p\mu(l_c^* - l_s)}{L(\mu)\left(\beta_s A(\tau - l_s) - \beta_c A(\tau - l_s)\right)} = \frac{L(\mu)\left(\beta_s A(\tau - l_s) - \beta_c A(\tau - l_c^*)\right)}{L(\mu)\left(\beta_s A(\tau - l_s) - \beta_c A(\tau - l_s)\right)} = \frac{\beta_s A(\tau - l_s) - \beta_c A(\tau - l_c^*)}{\beta_s A(\tau - l_s) - \beta_c A(\tau - l_s)}$$

Since $\dfrac{\partial A(\tau - l)}{\partial l} > 0$, we have $\beta_c A(\tau - l_c^*) \geq \beta_c A(\tau - l_s)$ which implies that $\dfrac{\widehat{\pi}^{oem}}{\widehat{\pi}^{total}} \leq 1$. The sensitivity results can be easily established by checking the first derivative and are omitted here for brevity. □

Proof of Proposition 4.(i)

The tree diagram below describes the possible set of borrowing scenarios under stochastic demand for $l=l_s$ and l_c representing the conventional external financing and reverse factoring respectively.

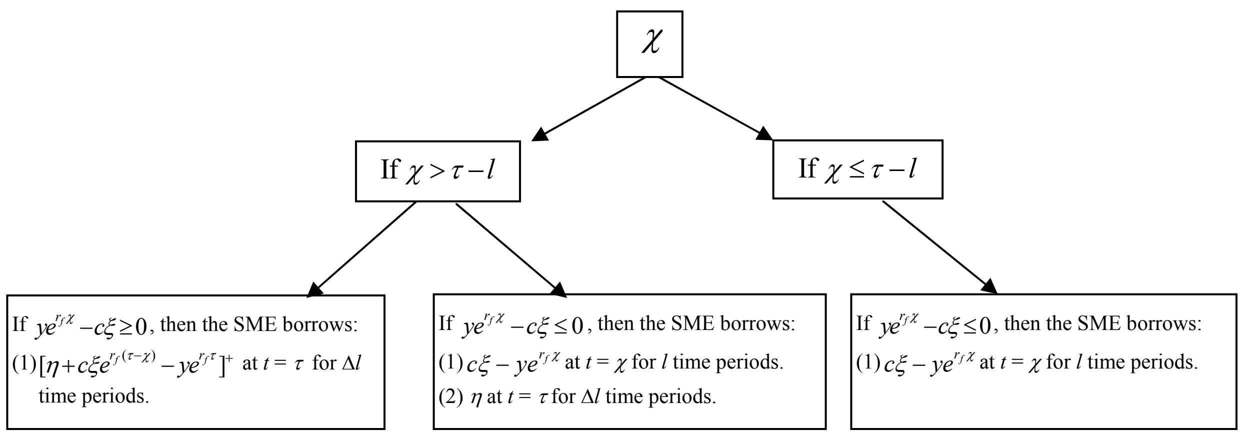

Figure 1A. SME's Borrowing Amount under Stochastic MTO

Then, for a given realization of stochastic parameters, following the developments in the proof of Theorem 1, the discounted value of the cash flows under conventional external financing is given by:

$$\begin{aligned}
V_{ext}^{sme}(.\mid\chi,\xi) &= p\xi e^{-r_f(\chi+l_s)} - c\xi e^{-r_f\chi} + \Big(\frac{(\eta + c\xi e^{r_f(\tau-\chi)} - ye^{r_f\tau})}{e^{r_f\tau}} - \frac{(\eta + c\xi e^{r_f(\tau-\chi)} - ye^{r_f\tau})e^{r_s\Delta l_s}}{e^{r_f(\chi+l_s)}}\Big) I_{\Omega_1^{l_s}} \\
&\quad + \Big(\frac{c\xi - ye^{r_f\chi}}{e^{r_f\chi}} - \frac{(c\xi - ye^{r_f\chi})e^{r_s l_s}}{e^{r_f(\chi+l_s)}}\Big) I_{\Omega_3} + \left(\frac{\eta}{e^{r_f\tau}} - \frac{\eta e^{r_s\Delta l_s}}{e^{r_f(\chi+l_s)}}\right) I_{\Omega_2^{l_s}},
\end{aligned}$$

$$\begin{aligned}
where\quad \Omega_1^{l_s} &= \{\xi,\chi : c\xi \leq ye^{r_f\chi}, \eta + c\xi e^{r_f(\tau-\chi)} - ye^{r_f\tau} \geq 0, \chi > \tau - l_s\}, \\
\Omega_2^{l_s} &= \{\xi,\chi : c\xi \geq ye^{r_f\chi}, \chi > \tau - l_s\}, \\
\Omega_3 &= \{\xi,\chi : c\xi \geq ye^{r_f\chi}, \chi \leq \tau - l_s\} \cup \{\xi,\chi : c\xi \geq ye^{r_f\chi}, \chi > \tau - l_s\} = \{\xi,\chi : c\xi \geq ye^{r_f\chi}\}.
\end{aligned}$$

Next, we separately approximate the indicator sets and the discounted cash flows with the first order Taylor series approximation with respect to (r_f, β_s) around (0, 0). Then, after taking the expected values over the random quantities, we obtain:

$$\hat{V}_{ext}^{sme} = E_{\xi,\chi}[p\xi(1-r_f(\chi+l_s)) - c\xi(1-r_f\chi) - \beta_s\Delta l_s(\eta+c\xi-y)I_{\hat{\Omega}_1^{l_s}} - \beta_s l_s(c\xi-y)I_{\hat{\Omega}_3} - \beta_s\Delta l_s\eta I_{\hat{\Omega}_2^{l_s}}].$$

$$\begin{aligned} \textit{where} \quad \hat{\Omega}_1^{l_s} &= \{\xi,\chi : c\xi \le y(1+r_f\chi), \eta + c\xi(1+r_f(\tau-\chi)) - y(1+r_f\tau) \ge 0, \chi > \tau - l_s\}, \\ \hat{\Omega}_2^{l_s} &= \{\xi,\chi : c\xi \ge y(1+r_f\chi), \chi > \tau - l_s\}, \\ \hat{\Omega}_3 &= \{\xi,\chi : c\xi \ge y(1+r_f\chi)\}. \end{aligned}$$

Similarly, the derivation for the case with reverse factoring is

$$\hat{V}_{rev}^{sme} = E_{\xi,\chi}[p\xi(1-r_f(\chi+l_c)) - c\xi(1-r_f\chi) - (\beta_c+b)\Delta l_c(\eta+c\xi-y)I_{\hat{\Omega}_1^{l_c}} - (\beta_c+b)l_c(c\xi-y)I_{\hat{\Omega}_3} - (\beta_c+b)\Delta l_c\eta I_{\hat{\Omega}_2^{l_c}}].$$

$$\begin{aligned} \textit{where} \quad \hat{\Omega}_1^{l_c} &= \{\xi,\chi : c\xi \le y(1+r_f\chi), \eta + c\xi(1+r_f(\tau-\chi)) - y(1+r_f\tau) \ge 0, \chi > \tau - l_c\}, \\ \hat{\Omega}_2^{l_c} &= \{\xi,\chi : c\xi \ge y(1+r_f\chi), \chi > \tau - l_c\}, \\ \hat{\Omega}_3 &= \{\xi,\chi : c\xi \ge y(1+r_f\chi)\}. \end{aligned}$$

Then, the expected benefit for the SME is given by $\pi^{sme} = \hat{V}_{rev}^{sme} - \hat{V}_{ext}^{sme}$:

$$\begin{aligned} &= E_{\xi,\chi}[p\xi(1-r_f(\chi+l_c)) - c\xi(1-r_f\chi) - (\beta_c+b)\Delta l_c(\eta+c\xi-y)I_{\hat{\Omega}_1^{l_c}} - (\beta_c+b)l_c(c\xi-y)I_{\hat{\Omega}_3} - (\beta_c+b)\Delta l_c\eta I_{\hat{\Omega}_2^{l_c}}] \\ &\quad - E_{\xi,\chi}[p\xi(1-r_f(\chi+l_s)) - c\xi(1-r_f\chi) - \beta_s\Delta l_s(\eta+c\xi-y)I_{\hat{\Omega}_1^{l_s}} - \beta_s l_s(c\xi-y)I_{\hat{\Omega}_3} - \beta_s\Delta l_s\eta I_{\hat{\Omega}_2^{l_s}}]. \end{aligned}$$

$$\begin{aligned} \text{where} \quad \hat{\Omega}_1^{l} &= \{\xi,\chi : \xi \le y(1+r_f\chi)/c, \xi \ge (y(1+r_f\tau)-\eta)/(c(1+r_f(\tau-\chi))), \chi > \tau - l\}, \\ \hat{\Omega}_2^{l} &= \{\xi,\chi : \xi \ge y(1+r_f\chi)/c, \chi > \tau - l\}, \\ \hat{\Omega}_3 &= \{\xi,\chi : \xi \ge y(1+r_f\chi)/c\}, \end{aligned}$$

for $l = l_c, l_s$.

After rearranging the terms and omitting the "^" for notational simplicity, we get:

$$\begin{aligned} \pi^{sme} &= \beta_s\left(E_{\xi,\chi}[(c\xi-y)l_s I_{\Omega_3} + (\eta+c\xi-y)\Delta l_s I_{\Omega_1^{l_s}} + \eta\Delta l_s I_{\Omega_2^{l_s}}]\right) \\ &\quad -(\beta_c+b)\left(E_{\xi,\chi}[(c\xi-y)l_c I_{\Omega_3} + (\eta+c\xi-y)\Delta l_c I_{\Omega_1^{l_c}} + \eta\Delta l_c I_{\Omega_2^{l_c}}]\right) - p\mu r_f(l_c - l_s). \end{aligned}$$

$$\begin{aligned} \text{where} \quad \Omega_1^{l} &= \{\xi,\chi : \xi \le y(1+r_f\chi)/c, \xi \ge (y(1+r_f\tau)-\eta)/(c(1+r_f(\tau-\chi))), \chi > \tau - l\}, \\ \Omega_2^{l} &= \{\xi,\chi : \xi \ge y(1+r_f\chi)/c, \chi > \tau - l\}, \\ \Omega_3 &= \{\xi,\chi : \xi \ge y(1+r_f\chi)/c\}, \end{aligned}$$

for $l = l_c, l_s$.

Letting $L(l) = E_{\xi,\chi}[(c\xi-y)lI_{\Omega_3} + (\eta+c\xi-y)\Delta lI_{\Omega_1^l} + \eta\Delta lI_{\Omega_2^l}]$ for $l = l_c, l_s$, we obtain

$$\pi^{sme} = \beta_s L(l_s) - (\beta_c+b)L(l_c) - p\mu r_f(l_c - l_s).$$

□

Proof of Proposition 4(ii)

Follows directly from the proof of Proposition 1(*ii*).

□

Proof of Proposition 4(iii)

Follows directly from Proposition 4(*i*) and (*ii*).

□

Proof of Proposition 5

Let us first consider the cash flows to the SME under the conventional external financing case. Note that the free cash reserves continuously earn the risk-free rate. Let Q_{ext} be the stocking decision of the SME. Then, if the demand arrives before $t = \tau - l_s$, then by assumption the SME does not need to borrow, otherwise the firm borrows, $L_{t=\tau} = \eta + (cQ_{ext} - y)e^{r_f \tau}$, at time τ and pays a stochastic amount D at time $t = \chi + l_s > \tau$. Under the risk-neutral measure, the loan is priced by the bank so that:

$$L_{t=\tau} = \frac{E^Q(D)}{e^{r_s \Delta l_s}},$$

where $\Delta l_s = (\chi + l_s - \tau)$ is the loan duration. Hence for a given realization of χ and ξ SME's cash flows discounted to t = 0, is given by:

$$\begin{aligned} V_{ext}^{sme}(.\,|\,\chi,\xi) &= \frac{p\min(Q_{ext},\xi)}{e^{r_f(\chi+l_s)}} - cQ_{ext} + \left(\frac{L_{t=\tau}}{e^{r_f\tau}} - \frac{E^Q[D]}{e^{r_f(\chi+l_s)}}\right) I_{\{\chi>\tau-l_s\}} \\ &= \frac{p\min(Q_{ext},\xi)}{e^{r_f(\chi+l_s)}} - cQ_{ext} + \left(\frac{L_{t=\tau}}{e^{r_f\tau}} - \frac{e^{r_s\Delta l_s}L_{t=\tau}}{e^{r_f(\chi+l_s)}}\right) I_{\{\chi>\tau-l_s\}} \qquad \left(L_{t=\tau} = \frac{E^Q[D]}{e^{r_s\Delta l_s}}\right) \\ &= \frac{p\min(Q_{ext},\xi)}{e^{r_f(\chi+l_s)}} - cQ_{ext} + (1 - e^{\beta_s \Delta l_s}) L_{t=0} I_{\{\chi>\tau-l_s\}} \qquad \left(L_{t=0} = \frac{L_{t=\tau}}{e^{r_f\tau}},\ \Delta l_s = \chi + l_s - \tau\right) \end{aligned}$$

Applying the first order Taylor series approximation with respect to (r_f, β_s) around (0, 0) we obtain:

$$\hat{V}_{ext}^{sme}(.\,|\,\chi,\xi) = p\min(Q_{ext},\xi)(1 - r_f(\chi + l_s)) - cQ_{ext} - \beta_s \Delta l_s (\eta + cQ_{ext} - y) I_{\{\chi>\tau-l_s\}}$$

Then, the expected value of the cash flows at t = 0:

$$\hat{V}_{ext}^{sme} = E_{\xi,\chi}[p\min(Q_{ext},\xi)(1 - r_f(\chi + l_s)) - cQ_{ext} - \beta_s \Delta l_s (\eta + cQ_{ext} - y) I_{\{\chi>\tau-l_s\}}].$$

$$\hat{V}_{ext}^{sme} = pE\min(Q_{ext},\xi)(1 - r_f(\bar{\chi} + l_s)) - cQ_{ext} - \beta_s L(Q_{ext}) A(\tau - l_s).$$

where $\bar{\chi} = E(\chi)$, $A(\tau - l_s) = E_\chi[\Delta l_s I_{\{\chi>\tau-l_s\}}]$ and $L(Q_{ext}) = \eta + cQ_{ext} - y$.

Letting $p_{ext} = p(1 - r_f(\bar{\chi} + l_s))$, $c_{ext} = c(1 + \beta_s A(\tau - l_s))$ and rearranging the terms, we get:

$$\hat{V}_{ext}^{sme} = p_{ext} E_\xi[\min(Q_{ext},\xi)] - c_{ext} Q_{ext} - \beta_s (\eta - y) A(\tau - l_s)$$

Then the optimization problem of the SME is:

$$\max_{Q_{ext}} \hat{V}_{ext}^{sme}(Q_{ext}) = \max_{Q_{ext}} p_{ext} E_\xi[\min(Q_{ext},\xi)] - c_{ext} Q_{ext} - \beta_s (\eta - y) A(\tau - l_s)$$

Now, it is trivial to observe that this problem is equivalent to a newsvendor problem with the optimal critical fractile $(p_{ext} - c_{ext})/p_{ext}$. Hence, the optimal quantities under conventional external financing and reverse factoring are, respectively, given by:

$$F(Q^*_{ext}) = \frac{p_{ext} - c_{ext}}{p_{ext}} \text{ and } F(Q^*_{rev}) = \frac{p_{rev} - c_{rev}}{p_{rev}},$$

where $p_{rev} = p(1 - r_f(\bar{\chi} + l_c))$, and $c_{rev} = c(1 + (\beta_c + b)A(\tau - l_c))$. This completes the proof. □

Proof of Proposition 6.(i)

Follows from Proposition 5.

□

Proof of Proposition 6.(ii)

The OEM receives goods at time $t = \chi$ from the SME and immediately collects revenues from sales, $w\min(Q_{ext}, \xi)$. However, the payment to the SME, $p\min(Q_{ext}, \xi)$, is delayed by the payment period. Accordingly, following the same developments in the previous proofs, after the first order Taylor series approximation the value functions under conventional external financing and reverse factoring becomes:

$$\hat{V}^{oem}_{ext} = E_{\xi,\chi}[\min(Q^*_{ext}, \xi)\left(w(1 - r_f\chi) - p(1 - r_f(\chi + l_s))\right)],$$
$$\hat{V}^{oem}_{rev} = E_{\xi,\chi}[\min(Q^*_{rev}, \xi)\left(w(1 - r_f\chi) - p(1 - r_f(\chi + l_c))\right)].$$

Evaluating the expectations:

$$\hat{V}^{oem}_{ext} = E_\xi \min(Q^*_{ext}, \xi)\left(w(1 - r_f\bar{\chi}) - p(1 - r_f(\bar{\chi} + l_s))\right),$$
$$\hat{V}^{oem}_{rev} = E_\xi \min(Q^*_{rev}, \xi)\left(w(1 - r_f\bar{\chi}) - p(1 - r_f(\bar{\chi} + l_c)\right).$$

Recalling that $p_{ext} = p(1 - r_f(\bar{\chi} + l_s)), p_{rev} = p(1 - r_f(\bar{\chi} + l_c))$ and defining $\bar{w} = w(1 - r_f\bar{\chi})$ we obtain:

$$\hat{V}^{oem}_{ext} = E\min(Q^*_{ext}, \xi)\left(\bar{w} - p_{ext}\right) \text{ and } \hat{V}^{oem}_{rev} = E\min(Q^*_{rev}, \xi)\left(\bar{w} - p_{rev}\right).$$

Accordingly, the expected benefit from the reverse factoring is determined by:

$$\pi^{oem} = \hat{V}^{oem}_{rev} - \hat{V}^{oem}_{ext} = E\min(Q^*_{rev}, \xi)\left(\bar{w} - p_{rev}\right) - E\min(Q^*_{ext}, \xi)\left(\bar{w} - p_{ext}\right).$$

□

Proof of Proposition 6.(iii)

Trivially follows from (*i*) and (*ii*).

□

Proof of Corollary 3.

Recall that OEC and PC can be, respectively, stated as

$$\Delta_{OEC} = \beta_s A(\tau - l_s)(1 - r_f(\bar{\chi} + l_c)) - (\beta_c + b)A(\tau - l_c)(1 - r_f(\bar{\chi} + l_s)) - r_f(l_c - l_s) \geq 0$$

$$\Delta_{PC} = p_{rev}E\min(Q^*_{rev}, \xi) - p_{ext}E\min(Q^*_{ext}, \xi) - c(Q^*_{rev} - Q^*_{ext}) + \beta_s L(Q^*_{ext})A(\tau - l_s) - (\beta_c + b)L(Q^*_{rev})A(\tau - l_c) \geq 0$$

It is easy to show that both Δ_{OEC} and Δ_{PC} are monotone decreasing in β_c and l_c. Hence, these exists unique frontiers such that $\Delta_{OEC} = 0$ and $\Delta_{PC} = 0$.

Now suppose we are on the unique frontier described by $\Delta_{OEC} = 0$, i.e., the OEC constraint is tight. This frontier can be equivalently described by $Q^*_{rev} = Q^*_{ext}$ or $c_{rev} p_{ext} = c_{ext} p_{rev}$. Substituting this condition into the SME's benefits, i.e., to Δ_{PC}, we obtain:

$\Delta_{PC} = p_{ext}(\frac{c_{rev} - c_{ext}}{c_{ext}})E\min(Q^*_{ext}, \xi) + (c_{ext} - c_{rev})Q^*_{ext} + (\eta - y)\left(\frac{c_{ext} - c_{rev}}{c}\right)$, which can be simplified as

$$\Delta_{PC} = [(\beta_c + b)(A(\tau - l_c) - \beta_s A(\tau - l_s)][p_{ext}E\min(Q^*_{ext}, \xi) - c_{ext}Q^*_{ext} + (y - \eta)\left(1 + \beta_s A(\tau - l_s)\right)]$$

Given that OEC is tight ($c_{rev} p_{ext} = c_{ext} p_{rev}$), it holds that

$c(1 + \beta_s A(\tau - l_s))p(1 - r_f(\overline{\chi} + l_c)) = c(1 + (\beta_c + b)A(\tau - l_c))p(1 - r_f(\overline{\chi} + l_s))$. Rearranging the terms:

$$\frac{1 + \beta_s A(\tau - l_s)}{1 + (\beta_c + b)A(\tau - l_c)} = \frac{(1 - r_f(\overline{\chi} + l_s))}{(1 - r_f(\overline{\chi} + l_c))} \geq 1 \qquad (l_c \geq l_s)$$

Consequently, this implies that $\beta_s A(\tau - l_s) \geq (\beta_c + b)A(\tau - l_c)$. Therefore, the first multiplier in the definition of Δ_{PC} is non-negative and the sign of Δ_{PC} is determined by the second term which is a constant. In particular, the PC is tighter than OEC (i.e., $\Delta_{PC} \leq 0$) if and only if

$$p_{ext}E\min(Q^*_{ext}, \xi) - c_{ext}Q^*_{ext} + (y - \eta)\left(1 + \beta_s A(\tau - l_s)\right) \geq 0 \text{ or}$$

$$y \geq \eta - \frac{p_{ext}E\min(Q^*_{ext}, \xi) - c_{ext}Q^*_{ext}}{1 + \beta_s A(\tau - l_s)} \doteq \hat{y}$$

Note that when the risk-free rate is zero, the first multiplier in the definition of Δ_{PC} is always equal to zero, i.e., $(\beta_c + b)(A(\tau - l_c) - \beta_s A(\tau - l_s) = 0$ and hence PC and OEC coincide.

Proof of Proposition 7.(i)

The optimal contract for the SME is obtained by maximizing the SME's benefits subject to the participation constraint of the OEM and non-negative bank fees:

$$\begin{aligned}
\hat{\pi}^{sme} = \max_{l_c, b} \pi^{sme} = \max_{l_c, b} \; & p_{rev}E\min(Q^*_{rev}, \xi) - p_{ext}E\min(Q^*_{ext}, \xi) - c(Q^*_{rev} - Q^*_{ext}) \\
& + \beta_s L(Q^*_{ext})A(\tau - l_s) - (\beta_c + b)L(Q^*_{rev})A(\tau - l_c) \\
s.t. \quad & E\min(Q^*_{rev}, \xi)\left(\overline{w} - p_{rev}\right) \geq E\min(Q^*_{ext}, \xi)\left(\overline{w} - p_{ext}\right) \\
& b \geq 0, l_c \geq l_s.
\end{aligned}$$

Rearranging and eliminating the constant terms, the optimization problem reduces to:

$$\max_{l_c,b}\ p_{rev}E\min(Q^*_{rev},\xi)-c_{rev}Q^*_{rev}-(\eta-y_0)(\beta_c+b)A(\tau\text{-}l_c)$$
$$s.t.\quad E\min(Q^*_{rev},\xi)\left(\overline{w}-p_{rev}\right)\geq E\min(Q^*_{ext},\xi)\left(\overline{w}-p_{ext}\right)$$
$$b\geq 0, l_c\geq l_s.$$

The first partial derivative of the objective with respect to *b*:

$$\frac{\partial\pi^{sme}}{\partial b}=p_{rev}\overline{F}(Q^*_{rev})\frac{\partial Q^*_{rev}}{\partial b}-cA(\tau\text{-}l_c)Q^*_{rev}-c_{rev}\frac{\partial Q^*_{rev}}{\partial b}-(\eta-y_0)A(\tau\text{-}l_c)$$
$$\frac{\partial\pi^{sme}}{\partial b}=\frac{\partial Q^*_{rev}}{\partial b}[p_{rev}\overline{F}(Q^*_{rev})-c_{rev}]-A(\tau\text{-}l_c)(cQ^*_{rev}+\eta-y_0)$$

Recalling that $p_{rev}\overline{F}(Q^*_{rev})-c_{rev}=0$ and $cQ^*_{rev}+\eta-y>0$, $\frac{\partial\pi^{sme}}{\partial b}=-A(\tau\text{-}l_c)(cQ^*_{rev}+\eta-y_0)<0$. This concludes that $b^*=0$.

Similarly the first partial with respect to l_c:

$$\frac{\partial\pi^{sme}}{\partial l_c}=-pr_fE\min(Q^*_{rev},\xi)+p_{rev}\overline{F}(Q^*_{rev})\frac{\partial Q^*_{rev}}{\partial l_c}-c(\beta_c+b)\frac{\partial A(\tau\text{-}l_c)}{\partial l_c}Q^*_{rev}-c_{rev}\frac{\partial Q^*_{rev}}{\partial l_c}-(\eta-y_0)(\beta_c+b)\frac{\partial A(\tau\text{-}l_c)}{\partial l_c}$$
$$\frac{\partial\pi^{sme}}{\partial l_c}=-pr_fE\min(Q^*_{rev},\xi)+[p_{rev}\overline{F}(Q^*_{rev})-c_{rev}]\frac{\partial Q^*_{rev}}{\partial l_c}-\frac{\partial A(\tau\text{-}l_c)}{\partial l_c}(\beta_c+b)[cQ^*_{rev}+\eta-y_0]$$

Recalling that $\frac{\partial A(\tau\text{-}l_c)}{\partial l_c}>0$ and $cQ^*_{rev}+\eta-y>0$ we obtain,

$\frac{\partial\pi^{sme}}{\partial l_c}=-pr_fE\min(Q^*_{rev},\xi)-\frac{\partial A(\tau\text{-}l_c)}{\partial l_c}(\beta_c+b)[cQ^*_{rev}+\eta-y_0]<0$. So, SME's benefits are maximized when the payment period is not extended, i.e., $l^*_c=l_s$. Finally, it is trivial to observe that the OEM's participation constraint is also satisfied when $l^*_c=l_s$ and $b^*=0$.

□

Proof of Proposition 7.(ii)

Checking the first partial derivative of π^{oem} with respect to *b*, it can be easily observed that the OEM's benefits are maximized when $b=0$.

From the proof of Proposition 7(*i*) we know that $\frac{\partial\pi^{sme}}{\partial l_c}<0$, then also observing $\pi^{sme}(l_c=l_s)>0$ and $\pi^{sme}(l_c=\infty)<0$ conclude that there is a unique $l_c=l'_c$ which makes the SME's participation constraint tight, i.e.,

$$p_{rev}E\min(Q^*_{rev},\xi)-p_{ext}E\min(Q^*_{ext},\xi)-c(Q^*_{rev}-Q^*_{ext})+\beta_sL(Q^*_{ext})A(\tau\text{-}l_s)$$
$$-(\beta_c+b)L(Q^*_{rev})A(\tau\text{-}l_c)=0$$

Hence the OEM cannot extend the payment period above l'_c and should choose the payment period such that $l_s \le l_c \le l'_c$. In addition checking the second derivative of the OEM's benefits:
$$\frac{\partial^2 \pi_{oem}}{\partial l_c^{\,2}} = \frac{\partial^2 Q_{rev}}{\partial l_c^{\,2}}(1-F(Q_{rev}))(\overline{w}-p_{rev}) - \frac{\partial Q_{rev}}{\partial l_c}\frac{\partial Q_{rev}}{\partial l_c} f(Q)(\overline{w}-p_{rev}) + 2pr_f \frac{\partial Q_{rev}}{\partial l_c}(1-F(Q_{rev})),$$
reveals that the benefits are not necessarily concave under a general demand distribution. In particular, we numerically observed that under a beta distribution, second derivative can be positive. Accordingly the optimal payment period of the OEM is given by
$$l_c^* = \underset{l_s \le l_c \le l'_c}{\arg\max}\{E\min(Q_{rev}^*,\xi)\left(\overline{w}-p_{rev}\right) - E\min(Q_{ext}^*,\xi)\left(\overline{w}-p_{ext}\right)\},$$

□

Proof of Proposition 7.(iii)

The total supply chain benefit is given by:
$$\begin{aligned}\pi^{total} &= p_{rev}E\min(Q_{rev}^*,\xi) - p_{ext}E\min(Q_{ext}^*,\xi) - c(Q_{rev}^* - Q_{ext}^*) + \beta_s L(Q_{ext}^*)A(\tau - l_s) - (\beta_c + b)L(Q_{rev}^*)A(\tau - l_c) \\ &\quad + E\min(Q_{rev}^*,\xi)\left(\overline{w}-p_{rev}\right) - E\min(Q_{ext},\xi)\left(\overline{w}-p_{ext}\right) \\ &= \overline{w}\left(E\min(Q_{rev}^*,\xi) - E\min(Q_{ext}^*,\xi)\right) - c(Q_{rev}^* - Q_{ext}^*) + \beta_s L(Q_{ext}^*)A(\tau - l_s) - (\beta_c + b)L(Q_{rev}^*)A(\tau - l_c).\end{aligned}$$
And the optimization problem becomes:
$$\begin{aligned}\max_{l_c,b}\ \pi^{total} &= \overline{w}\left(E\min(Q_{rev}^*,\xi) - E\min(Q_{ext}^*,\xi)\right) - c(Q_{rev}^* - Q_{ext}^*) + \beta_s L(Q_{ext}^*)A(\tau - l_s) - (\beta_c + b)L(Q_{rev}^*)A(\tau - l_c) \\ s.t.\quad & p_{rev}E\min(Q_{rev}^*,\xi) - p_{ext}E\min(Q_{ext}^*,\xi) - c(Q_{rev}^* - Q_{ext}^*) + \beta_s L(Q_{ext}^*)A(\tau - l_s) - (\beta_c + b)L(Q_{rev}^*)A(\tau - l_c) \ge 0, \\ & E\min(Q_{rev}^*,\xi)\left(\overline{w}-p_{rev}\right) \ge E\min(Q_{ext}^*,\xi)\left(\overline{w}-p_{ext}\right), \\ & b \ge 0, l_c \ge l_s.\end{aligned}$$
Then, $\dfrac{\partial \pi^{total}}{\partial b} = \dfrac{\partial Q_{rev}^*}{\partial b}\left(\overline{w}\left(1-F(Q_{rev}^*)\right) - c_{rev}\right) - L(Q_{rev}^*)A(\tau - l_c).$

Recall that $p_{rev}\left(1-F(Q_{rev}^*)\right) - c_{rev} = 0$, $\overline{w} \ge p_{rev}$ and $\dfrac{\partial Q_{rev}^*}{\partial b} < 0.$ This concludes that $\dfrac{\partial \pi^{total}}{\partial b} < 0$ and $b^* = 0.$

Similarly, $\dfrac{\partial \pi^{total}}{\partial l_c} = \dfrac{\partial Q_{rev}^*}{\partial l_c}\left(\overline{w}\left(1-F(Q_{rev}^*)\right) - c_{rev}\right) - (\beta_c + b)L(Q_{rev}^*)\dfrac{\partial A(\tau - l_c)}{\partial l_c}.$

Recalling that $p_{rev}\left(1-F(Q_{rev}^*)\right) - c_{rev} = 0$, $\dfrac{\partial Q_{rev}^*}{\partial l_c} < 0$, $\dfrac{\partial A(\tau - l_c)}{\partial l_c} > 0$, $\overline{w} \ge p_{rev}$, and observing $\overline{w}\left(1-F(Q_{rev}^*)\right) - c_{rev} \ge 0$, we conclude that $\dfrac{\partial \pi^{total}}{\partial l_c} < 0$. Hence, $l_c^* = l_s$.

From the proof of the Proposition 7 (*i*) and (*ii*) we know that the participation constraints of the SME and the OEM are also satisfied at this solution.

□